Unit 1: The Concept of Rate and Algebraic Thinking

This unit emphasizes how the concept of rate is important in mathematics and the everyday world. We will consider the rate of speed in this unit and we'll look at other rates in later units.

In this unit, you will discover:

- how your knowledge of rate can help you with algebra
- how algebra can help you apply your knowledge of rate.

Unit Focus

Number Sense, Concepts, and Operations

- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand and use the real number system. (MA.A.2.4.2)
- Understand and explain the effects of addition, subtraction, multiplication, and division on real numbers, including square roots, exponents, and appropriate inverse relationships. (MA.A.3.4.1)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)

Measurement

- Use concrete and graphic models to derive formulas for finding rate, distance, time, and angle measures. (MA.B.1.4.2)

- Solve real-world problems involving rated measures (miles per hour, feet per second). (MA.B.2.4.2)
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)

Algebraic Thinking

- Describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, tables, and graphs. (MA.D.1.4.1)
- Determine the impact when changing parameters of given functions. (MA.D.1.4.2)
- Represent real-world problem situations using finite graphs. (MA.D.2.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)

Data Analysis and Probability

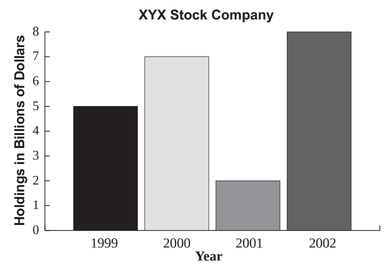
- Interpret data that has been collected, organized, and displayed in charts, tables, and plots. (MA.E.1.4.1)
- Calculate measures of central tendency (mean, median, and mode) and dispersion (range) for complex sets of data and determine the most meaningful measure to describe the data. (MA.E.1.4.2)
- Design and perform real-world statistical experiments, then analyze results and report findings. (MA.E.3.4.1)

Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

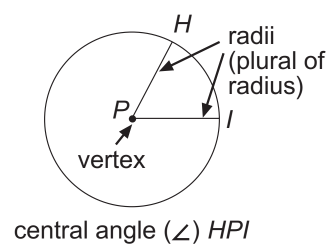
axes (of a graph) .... the horizontal and vertical number lines used in a coordinate plane system; (singular: *axis*)

bar graph .... a graph that uses either vertical or horizontal bars to display data

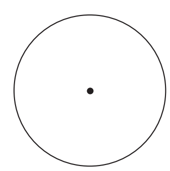


center (of a circle) .... the point from which all points on the circle are the same distance

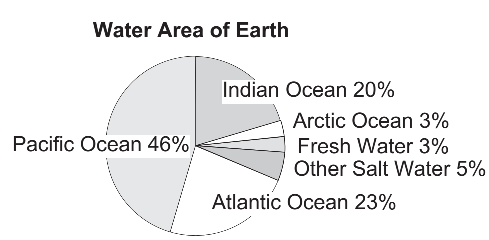
central angle (of a circle).............. an angle that has its vertex at the center of a circle, with radii as its sides



circle .... the set of all points in a plane that are all the same distance from a given point called the center



circle graph .... a data display that divides a circle into regions representing a portion of the total set of data; the circle represents the whole set of data



coordinate grid or plane .... a two-dimensional network of horizontal and vertical lines that are parallel and evenly-spaced; especially designed for locating points, displaying data, or drawing maps

coordinate plane .... the plane containing the x- and y-axes

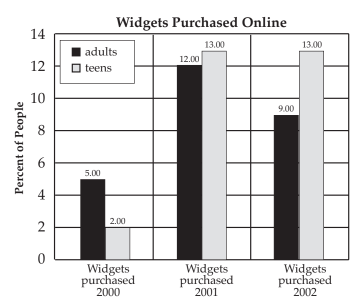
coordinates .... numbers that correspond to points on a coordinate plane in the form (x, y), or a number that corresponds to a point on a number line

data .... information in the form of numbers gathered for statistical purposes

data displays/graphs .... different ways of displaying data in charts, tables, or graphs
Example: pictographs, circle graphs, single-, double-, or triple-bar and line graphs, histograms, stem-and-leaf plots, box-and-whisker plots, and scatterplots

degree (°) .... common unit used in measuring angles

double bar graph .... a graph used to compare quantities of *two* sets of data in which length of bars are used to compare numbers



equation .... a mathematical sentence in which two expressions are connected by an equality symbol
Example: $2x = 10$

formula .... a way of expressing a relationship using variables or symbols that represent numbers

fraction .... any part of a whole
Example: One-half written in fractional form is $\frac{1}{2}$.

graph .... a drawing used to represent data
Example: bar graphs, double bar graphs, circle graphs, and line graphs

graph of a point .... the point assigned to an ordered pair on a coordinate plane

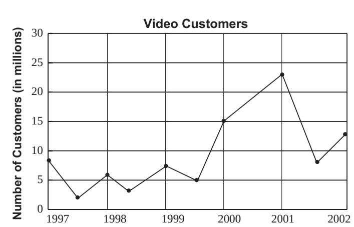
intersection .... the point at which lines or curves meet; the line where planes meet

labels (for a graph) .... the titles given to a graph, the axes of a graph, or the scales on the axes of a graph

linear equation .... an algebraic equation in which the variable quantity or quantities are raised to the zero or first power and the graph is a straight line
Example: $20 = 2(w + 4) + 2w$; $y = 3x + 4$

linear relationship .... relationships between two variables that can be expressed as straight-line graphs

line graph .... a graph that displays data using connected line segments



line segment (—) .... a portion of a line that consists of two defined endpoints and all the points in between
Example: The line segment *AB* is between point *A* and point *B* and includes point *A* and point *B*.

maximum .... the largest amount or number allowed or possible

mean (or average) .... the arithmetic average of a set of numbers; a measure of central tendency

measures of central tendency ..... the mean, median, and mode of a set of data

median .... the middle point of a set of rank-ordered numbers where half of the numbers are above the median and half are below it; a measure of central tendency

minimum .... the smallest amount or number allowed or possible

mode .... the score or data point found most often in a set of numbers; a measure of central tendency
Example: There may be no mode, one mode, or more than one mode in a set of numbers.

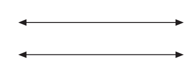
nonlinear equation .... an equation whose graph is *not* a line

ordered pair .... the location of a single point on a rectangular coordinate system where the first and second values represent the position relative to the x-axis and y-axis, respectively
Example: (x, y) or $(3, -4)$

order of operations .... the order of performing computations in parentheses first, then exponents or powers, followed by multiplication and/or division (as read from left to right), then addition and/or subtraction (as read from left to right); also called *algebraic order of operations*
Examples: $5 + (12 - 2) \div 2 - 3 \times 2 =$
$5 + 10 \div 2 - 3 \times 2 =$
$5 + 5 - 6 =$
$10 - 6 =$
4

origin .... the point of intersection of the x- and y-axes in a rectangular coordinate system, where the x-coordinate and y-coordinate are both zero (0)

parallel (||) .... being an equal distance at every point so as to never intersect



parallel lines .... two lines in the same plane that are a constant distance apart; lines with equal slopes

percent (%) .... a special-case ratio which compares numbers to 100 (the second term)
Example: 25% means the ratio of 25 to 100.

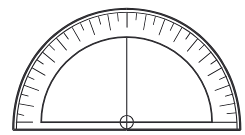
point .... a specific location in space that has no discernable length or width

positive numbers .... numbers greater than zero

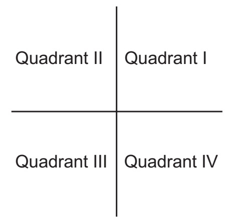
power (of a number) .... an exponent; the number that tells how many times a number is used as a factor
Example: In 2^3, 3 is the power.

product .... the result of multiplying numbers together
Example: In 6 x 8 = 48,
48 is the product.

protractor .... an instrument used for measuring and drawing angles



quadrant .... any of four regions formed by the axes in a rectangular coordinate system



quotient .... the result of dividing two numbers
Example: In 42 ÷ 7 = 6,
6 is the quotient.

range (of a set of numbers) .... the lowest value (L) in a set of numbers through the highest value (H) in the set
Example: When the width of the range is expressed as a single number, the range is calculated as the difference between the highest and the lowest values (H – L). Other presentations show the range calculated as H – L + 1. Depending upon the context, the result of either calculation would be considered correct.

rate/distance .... calculations involving rates, distances, and time intervals, based on the distance, rate, time formula ($d = rt$); a ratio that compares two quantities of different units
Example: feet per second

rate of change .... how a quantity is changing over time

root .... an equal factor of a number

Example:

In $\sqrt{144} = 12$, 12 is the square root.

In $\sqrt[3]{125} = 5$, 5 is the cube root.

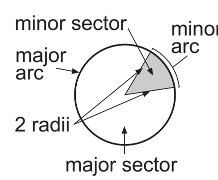
rounded number .... a number approximated to a specified place

Example: A commonly used rule to round a number is as follows.

- If the digit in the first place after the specified place is 5 or more, *round up* by adding 1 to the digit in the specified place ($\underline{4}61$ rounded to the nearest hundred is 500).
- If the digit in the first place after the specified place is less than 5, *round down* by *not* changing the digit in the specified place ($\underline{4}41$ rounded to the nearest hundred is 400).

scale .... the numeric values, set at fixed intervals, assigned to the axes of a graph

sector .... a part of a circle bounded by two radii and the arc or curve created between any two of its points



slope .... the ratio of change in the vertical axis (y-axis) to each unit change in the horizontal axis (x-axis) in the form $\frac{\text{rise}}{\text{run}}$ or $\frac{\Delta y}{\Delta x}$; the constant, m, in the linear equation for the slope-intercept form $y = mx + b$

solution .... any value for a variable that makes an equation or inequality a true statement
Example: In $y = 8 + 9$
$y = 17$ 17 is the solution.

standard form .... a method of writing the common symbol for a numeral
Example: The standard numeral for five is 5.

stem-and-leaf plot .... a graph that organizes data by place value to compare data frequencies

Number of Goals Scored

Stem	Leaves
1	5 9
2	1 3 7 7 7
3	0 1 3 4 4 5 6 7
4	2 3 6 7 8

Key: 2 | 3 represents 23.

sum .... the result of adding numbers together
Example: In $6 + 8 = 14$,
14 is the sum.

table (or chart) .... a data display that organizes information about a topic into categories

variable .... any symbol, usually a letter, which could represent a number

whole number .... the numbers in the set {0, 1, 2, 3, 4, …}

***x*-axis** .... the horizontal number line on a rectangular coordinate system

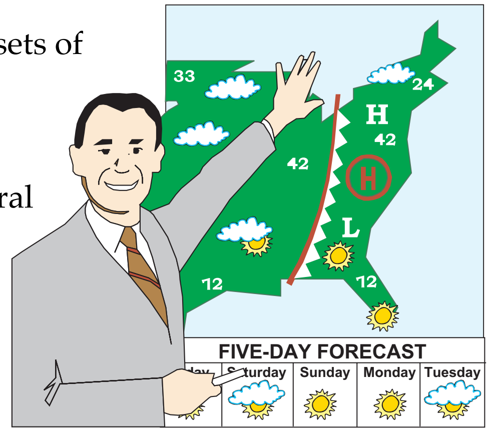
***x*-coordinate** .... the first number of an ordered pair

***y*-axis** .... the vertical number line on a rectangular coordinate system

***y*-coordinate** .... the second number of an ordered pair

Unit 1: The Concept of Rate and Algebraic Thinking

Introduction



In analyzing data, we learn to organize sets of statistics in ways that let us make better decisions. The various forms of media which provide us with information frequently use graphs, measures of central tendency (mean, median, and mode), and probability. Television weather reporters can predict for a week at a time the probability of rain for a given geographic area.

Therefore, in order to keep up with current events, a person needs to be able to formulate hypotheses; collect and interpret data; and draw conclusions based on statistics, tables, graphs, and charts. Furthermore, we need to recognize ways in which statistics can be misleading. Clever statisticians can devise graphs or charts that are deceptive. A person with a good knowledge of data analysis usually develops good skills for interpreting and evaluating the accuracy or inaccuracy of statistical presentations.

- You will be working with *variables*, or symbols that represent numbers, and algebraic expressions.
- You will be learning different ways to analyze and express patterns, relations, and functions, including words, tables, graphs, geometric formulas, and linear equations and inequalities.
- You will be working with relations expressed by ratios, rates, and proportions.
- You will have many opportunities to become skillful in using a calculator.

Mastery of these skills is essential in our ever-changing technical world. Proficiency in algebra will help you develop skills in abstract thinking required to solve a variety of problems.

Lesson One Purpose

- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use concrete and graphic models to derive formulas for finding rate, distance, time, and angle measures. (MA.B.1.4.2)
- Solve real-world problems involving rated measures (miles per hour, feet per second). (MA.B.2.4.2)
- Describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, tables, and graphs. (MA.D.1.4.1)
- Interpret data that has been collected, organized, and displayed in charts, tables, and plots. (MA.E.1.4.1)
- Calculate measures of central tendency (mean, median, and mode) and dispersion (range) for complex sets of data and determine the most meaningful measure to describe the data. (MA.E.1.4.2)

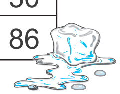
Data Analysis

There are many types of **data displays** used to organize and visually compare information. You can use **graphs**, **tables**, **or charts** to find or compare information. A *table* is a data display that organizes information about a topic into categories.

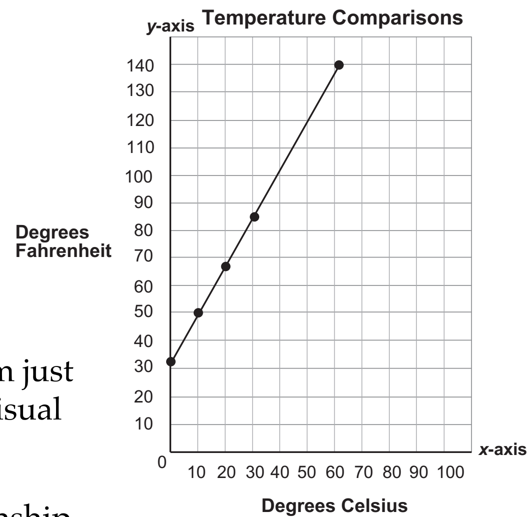
Your science book may have a table very similar to the one below.

Temperature Comparisons

C	0	10	20	30
F	32	50	68	86



The *graph* to the right is a visual representation of the **data** from the table above. The graph is called a **line graph**. A *line graph* is a graph that displays data using connected **line segments (—)**. The *data* used is information in the form of numbers gathered for statistical purposes. A line graph allows a person to see patterns that may not be obvious from just an **equation**. A graph is a powerful visual tool for representing data.



The table and graph show the relationship between Fahrenheit and Celsius temperatures. For example, a temperature of 0 degrees Celsius corresponds to a temperature of 32 degrees Fahrenheit, the temperature at which water freezes.

Exploring Data Using a Stem-and-Leaf Plot

A **stem-and-leaf plot** organizes data by place value to compare data frequencies. It is a quick way to picture the shape of data while including the actual numerical values in the *data display* or graph.

- The *stem* is the number to the left of a vertical (↕) line in the display. In the stem-and-leaf plot below, the stem represents the *tens digit* for each data entry.
- The *leaves* in the plot are to the right of the vertical line, and they represent the *final digit* in the number. The leaves in our plot below represent the *units digit* for each number.

Stem	Leaves
2	1 1 3 7 8

In this example, the data entries are 21, 21, 23, 27, and 28.

- What the stem and leaf represent are explained in the key.

Key:	**2**	3 = 23

Describing Data Using Measures of Central Tendency and Range

What if you wanted to find the average of a long list of numbers? The stem-and-leaf plot is also a convenient way to organize data to determine the **mean**, **median**, **mode**, and **range** for the distribution.

The *mean*, *median*, and *mode* are sometimes called **measures of central tendency**. *Measures of central tendency* describe how *data* are *centered*. Each of these measures describes a set of data in a slightly different way. The **range** (of a set of numbers) is the difference between the highest and lowest value in a set of data. The *range* can help you decide if the differences among the data are important.

Example of range: 100 – 0 = 100

The **mean (or average)** is the **sum** of the data divided by the number of items.

Example of mean: $\frac{0 + 55 + 70 + 80 + 80 + 82 + 85 + 88 + 100}{9} = \frac{640}{9} = 71$ rounded

When the data are centered around the *mean*, it is an appropriate measure of central tendency. The mean can be distorted by an *extreme* value, a value that is much greater than or less than the other values.

The **median** is the middle item when the data are listed in numerical order. (If there is an even number of items, the median is the average of the two middle numbers.)

Example of median: 0, 55, 70, 80, **80**, 82, 85, 88, 100

When there is an extreme value in a set of data that distorts the mean, the *median* is an appropriate measure of central tendency.

The **mode** is the item that appears more often in a set of numbers. There can be more than one mode. There also can be *no* mode if each item appears only once.

Example of mode: 0, 55, 70, **80**, **80**, 82, 85, 88, 100

When data cannot be averaged (to find a mean) or listed in numerical order (to find a median), the *mode* is the appropriate measure of central tendency.

Data Analysis with Speed Limits

The following numbers show the maximum posted speed limits (in miles per hour—mph) for passenger vehicles as of January 2003 for the 50 states and the District of Columbia:

Rural Interstates
60 65 65 65 65 65 65 65 65 65 65 65 65 65 65 65 65 65 65 65 65 75 75 75 75 75
75 75 75 75 75 75 70 70 70 70 70 70 70 70 70 70 70 70 70 70 70 70 70 70 70

Urban Interstates
50 55 55 55 55 55 55 55 55 55 55 55 55 55 55 55 55 55 55 55 55 55 60 60 60 65
65 65 65 65 65 65 65 65 65 65 65 65 65 65 65 65 65 65 70 70 70 70 70 70 70

Note: The District of Columbia has no rural interstates. Urban sections fall within a census area with an urban population of 5,000 to 49,999 or within a designated urbanized area with a population of 50,000 or greater.

The *double stem-and-leaf plot* (also called the *back-to-back stem-and-leaf plot*) is a convenient way to compare two sets of data. The double stem-and-leaf plot below compares the posted speed limits on the two types of interstates, rural and urban.

- We see that no state has a speed limit below 60 on *rural* interstates since there are no *leaves* denoting units digits to the left of the *stem*, 5.
- To the right of the stem, 5, we see that one state has a speed limit of 50 mph, and 21 states have a speed limit of 55 mph on *urban* interstates.

Speed Limits

Rural Interstates Leaves	Stem	Urban Interstates Leaves
	5	0555555555555555555555
55555555555555555555 0	**6**	0005555555555555555555
55555555555000000000000000000	**7**	0000000

Key:			
	5	0	means 50 mph
0	**6**		means 60 mph

Practice

Use the **double stem-and-leaf plot** *below to complete the following.*

Speed Limits

Rural Interstates Leaves	Stem	Urban Interstates Leaves
	5	0555555555555555555555
555555555555555555550	**6**	0005555555555555555555
55555555555000000000000000000	**7**	0000000

Key:			
	5	0	means 50 mph
0	**6**		means 60 mph

1. Find the *range*, the difference between the highest and lowest value.

   a. The range for rural interstates is ______ to ______ or ______ .

   b. The range for urban interstates is ______ to ______ or ______ .

2. Find the *median*, the middle number when the data is listed in numerical order.

   a. The median for rural interstates is ______ .

   b. The median for urban interstates is ______ .

3. Find the *mode*, the number that appears most often in a set of numbers.

   a. The mode for rural interstates is ______ .

   b. The mode for urban interstates is ______ .

4. Find the *mean*, the sum of the numbers divided by the number of items.

   a. The mean for rural interstates, **rounded** to the nearest **whole number**, is ______ .

   b. The mean for urban interstates, **rounded** to the nearest **whole number**, is ______ .

5. By examining the double stem-and-leaf plot on the previous page, the number of states that have the *same* speed limit can be determined. Complete the following.

   ______ states have a 60 mph speed limit on rural interstates.

   ______ states have a 65 mph speed limit on rural interstates.

   ______ states have a 70 mph speed limit on rural interstates.

   ______ states have a 75 mph speed limit on rural interstates.

6. Which of the following methods do you prefer for finding the *mean*?

Method One

(60 + 65 + 65 + 65 + 65 + 65 + 65 + 65 + 65 + 65 + 65 + 65 + 65 + 65 + 65 + 65 + 65 + 65 + 65 + 65 + 65 + 70 + 70 + 70 + 70 + 70 + 70 + 70 + 70 + 70 + 70 + 70 + 70 + 70 + 70 + 70 + 70 + 70 + 70 + 75 + 75 + 75 + 75 + 75 + 75 + 75 + 75 + 75 + 75 + 75) ÷ 50 =

Method Two

[1(60) + 20(65) + 18(70) + 11(75)] ÷ 50 =

I prefer Method ______ because ________________________________

____ .

Note: You may have experienced buying several of the same items at the grocery store. Sometimes the cashier may have scanned one and entered the number of the items being purchased. Sometimes the cashier may have scanned each one individually. Consider the time required for each method and which might be to the advantage of the customer as well as the store.

7. For the problem below, use a **four-function calculator** that does *not* have the **order of operations** in its memory, only the plus, minus, multiplication, and division sign. Enter the problem from Method Two into the calculator just as you see it below.

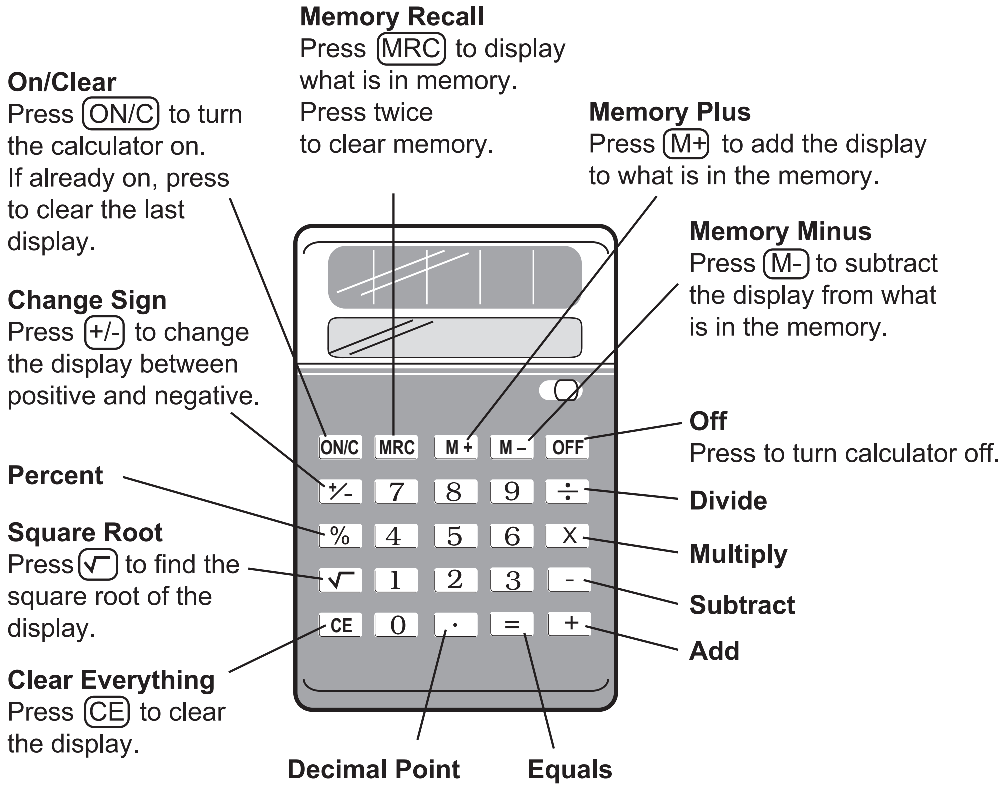
1 x 60 + 20 x 65 + 18 x 70 + 11 x 75 ÷ 50 = ____________ mph

When you see the result, you will know an *error* has been made because the answer is *very unreasonable*! The average speed limit for rural interstates is not even close to a half-million miles per hour! This calculator simply does the operations in the order in which they are entered. See below.

1 x 60 = 60
60 + 20 = 80
80 x 65 = 5200
5200 + 18 = 5218
5218 x 70 = 365260
365260 + 11 = 365271
365271 x 75 = 27395325
27395325 ÷ 50 = 547906.5 mph   unreasonable answer

The calculator below shows functions you might find on your *four-function calculator*.

An Example of a Four-Function Calculator



8. Use a **scientific calculator** with the *order of operations** in its memory and enter the following:

$1 \times 60 + 20 \times 65 + 18 \times 70 + 11 \times 75 \div 50 =$ ____________ mph

When you see the result, you will know an *error* has been made because this answer is also *unreasonable*.

The calculator used the correct order of operations for what we entered but we need to consider what we entered:

$1 \times 60 + 20 \times 65 + 18 \times 70 + 11 \times 75 \div 50 =$

The calculator found **products** and **quotients** as they appeared left to right, as follows:

$60 + 1300 + 1260 + 825 \div 50 =$
$60 + 1300 + 1260 + 16.5 =$
2636.50 unreasonable answer

*Check your calculator for order of operations. Enter the following:

$18 + 9 \div \sqrt{9} =$

Remember: To enter $\sqrt{9}$, first press 9 then press $\sqrt{\ }$.

If the answer is 21, your calculator has order of operations. If the answer is 9, it does *not*.

If your calculator does *not* have order of operations, then you will need to find the answer to $\sqrt{9}$ to use in the equation.

- First enter 9 [$\sqrt{\ }$] which gives you the answer of 3.
- Then press [CE] to clear everything.
- Now enter 9 [÷] 3 [+] 18 [=] and you should get the answer 21.

9. Use a *scientific calculator* with the order of operations in its memory. Enter the following, using the grouping symbols ( ).

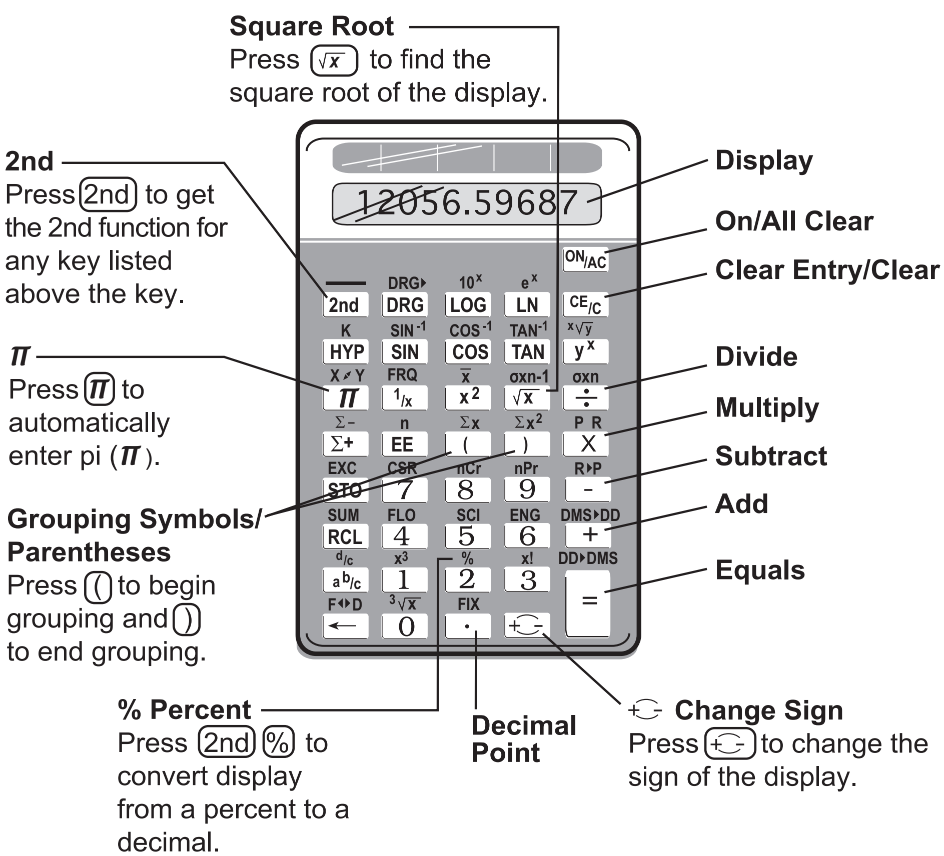
   When you enter [( ], the calculator waits until you enter [ )] before calculating what is between the grouping symbols.

   (1 x 60 + 20 x 65 + 18 x 70 + 11 x 75) ÷ 50.

   This tells the calculator to *divide the* sum *of the four products* by 50 and yields the correct answer.

The calculator below shows functions you might find on your scientific calculator.

An Example of a Scientific Calculator



Order of Operations

Consider the following.

Evaluate $5 + 4 \bullet 3 =$

Is the answer 27 or is the answer 17? You could argue that both answers are valid, although 17 is the universally accepted answer. Mathematicians have agreed on the following *order of operations*.

Rules for Order of Operations

Always start on the *left* and move *to* the *right*.

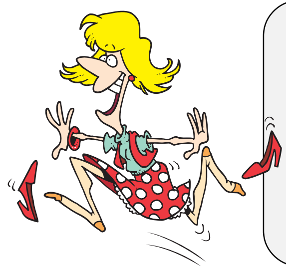
1.	Do operations inside *parentheses* first.	( ), [ ], $\frac{x}{y}$
2.	Then do all *powers* (exponents) **or** *roots*.	x^2 **or** $\sqrt{x}$
3.	Next do *multiplication* **or** *division*— as they occur from left to right.	x **or** ÷
4.	Finally, do *addition* **or** *subtraction*— as they occur from left to right.	+ **or** –

Note: The fraction bar sometimes comes in handy to show grouping.

Example: $\frac{3x^2 + 8}{2} = (3x^2 + 8) \div 2$

The *order of operations* makes sure everyone doing the problem will get the same answer.

Some people remember these rules by using this mnemonic device to help their memory.



Please **P**ardon **M**y **D**ear **A**unt **S**ally

Please .... **P**arentheses
Pardon .... **P**owers
My **D**ear .... **M**ultiplication or **D**ivision
Aunt **S**ally .... **A**ddition or **S**ubtraction

Remember: You do multiplication **or** division—*as they occur* from *left to right*—and then addition **or** subtraction—*as they occur* from *left to right*.

Study the following.

$$25 - 3 \bullet 2 =$$

There are no parentheses. There are no *powers* or *roots*. We look for multiplication or division and find multiplication. We multiply. We look for addition or subtraction and find subtraction. We subtract.

$$\begin{aligned} 25 - 3 \bullet 2 &= \\ 25 - 6 &= \\ 19 & \end{aligned}$$

Study the following.

$$12 \div 3 + 6 \div 2 =$$

There are no parentheses. There are no powers or roots. We look for multiplication or division and find division. We divide. We look for addition or subtraction and find addition. We add.

$$\begin{aligned} 12 \div 3 + 6 \div 2 &= \\ 4 + 3 &= \\ 7 & \end{aligned}$$

If the rules were ignored, one might divide 12 by 3 and get 4, then add 4 and 6 to get 10, then divide 10 by 2 to get 5. That is why agreement is needed, and why we use the agreed-upon *order of operations*.

Study the following.

$30 - 3^3 =$

There are no parentheses. We look for powers and roots and find powers, 3^3. We calculate this. We look for multiplication or division and find none. We look for addition or subtraction and find subtraction. We subtract.

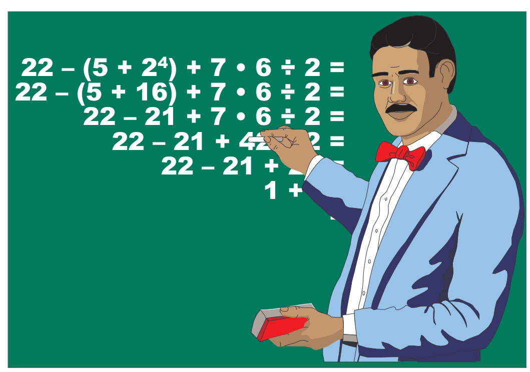
$$
\begin{aligned}
30 - 3^3 &= \\
30 - 27 &= \\
3
\end{aligned}
$$

Study the following.

$22 - (5 + 2^4) + 7 \bullet 6 \div 2 =$

We look for parentheses and find them. We must do what is inside the parentheses first. We find addition and a power. We do the power first and then the addition. There are no roots. We look for multiplication or division and find both. We do them in the order they occur, left to right, so the multiplication occurs first. We look for addition or subtraction and find both. We do them in the order they occur, left to right, so the subtraction occurs first.

$$
\begin{aligned}
22 - (5 + 2^4) + 7 \bullet 6 \div 2 &= \\
22 - (5 + 16) + 7 \bullet 6 \div 2 &= \\
22 - 21 + 7 \bullet 6 \div 2 &= \\
22 - 21 + 42 \div 2 &= \\
22 - 21 + 21 &= \\
1 + 21 &= \\
22
\end{aligned}
$$



Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. a drawing used to represent data

______ 2. a graph that organizes data by place value to compare data frequencies

______ 3. different ways of displaying data in charts, tables, or graphs

______ 4. a data display that organizes information about a topic into categories

______ 5. a graph that displays data using connected line segments

______ 6. information in the form of numbers gathered for statistical purposes

______ 7. an equal factor of a number

______ 8. a mathematical sentence in which two expressions are connected by an equality symbol

______ 9. an exponent; the number that tells how many times a number is used as a factor

______ 10. a number approximated to a specified place

A. data

B. data displays

C. equation

D. graph

E. line graph

F. power (of a number)

G. root

H. rounded number

I. stem-and-leaf plot

J. table (or chart)

Practice

Use the list below to write the correct term for each definition on the line provided.

mean	**mode**	**quotient**
measures of central tendency	**order of operations**	**range**
median	**product**	**sum**

_________________________ 1. the middle point of a set of ordered numbers where half of the numbers are above the median and half are below it

_________________________ 2. the order of performing computations in parentheses first, then exponents or powers, followed by multiplication and/or division (as read from left to right), then addition and/or subtraction (as read from left to right)

_________________________ 3. the mean, median, and mode of a set of data

_________________________ 4. the lowest value (L) in a set of numbers through the highest value (H) in the set

_________________________ 5. the score or data point found most often in a set of numbers

_________________________ 6. the arithmetic average of a set of numbers

_________________________ 7. the result of adding numbers together

_________________________ 8. the result of dividing two numbers

_________________________ 9. the result of multiplying numbers together

Finding Distance Traveled

What would you need to know to find the *distance* traveled by a person, vehicle, or object? You would need to know the following:

- the **rate** or average speed and
- the amount of time spent traveling.

The **formula** $d = rt$ can be used to find the distance traveled (d), if the rate (r) and the amount of time (t) are known.

- To find the distance traveled, we multiply the rate of speed by the time.

distance = rate x time

Example of finding distance traveled:

If the posted speed limit on a highway is 65 miles per hour (mph) and I maintain that speed for 1 hour, I will have traveled 65 miles.

$d = rt$
$d = 65(1)$
$d = 65$

If I maintain that speed for 2 hours, I will have traveled 130 miles.

$d = rt$
$d = 65(2)$
$d = 130$

Practice

Use the **distance formula** *below to complete the following table.*

distance = rate x time
$d = rt$

Distance in Miles

Time (in hours)	Rate (in miles per hour)	Distance (in miles)
1	50	
1	55	
1	60	
1	65	
1	70	
1	75	
4	50	
4	55	
4	60	
4	65	
4	70	
4	75	
8	50	
8	55	
8	60	
8	65	
8	70	
8	75	

Finding Rate or Average Speed

What would you do if you knew the distance and time traveled but needed to find the rate or average speed?

- The formula for distance is $d = rt$. You can find the rate by dividing the distance by the time as follows:

$$\text{rate} = \frac{\text{distance}}{\text{time}}$$

Example of finding rate or average speed:

If I travel 240 miles in 6 hours, my average rate of speed can be found by dividing the distance, 240, by the time, 6. An equivalent equation or formula for finding rate is as follows:

$$r = \frac{d}{t}$$
$$r = \frac{240}{6}$$
$$r = 40$$

We could also use the original formula, $d = rt$, and substitute 240 for d and 6 for t.

$$d = r(t)$$
$$240 = r(6)$$

To solve this equation intuitively, I can ask: what do I multiply 6 by to get 240?

6 times ? = 240
6 times 40 = 240 $\quad$ $40 = r$

Here is one way to work the problem.

I can divide both sides of the equation by 6 to solve it in a step-by-step symbolic manner.

$$d = rt$$
$$240 = 6r$$
$$\frac{240}{6} = \frac{6r}{6} \qquad \text{divide each side of the equation by 6}$$
$$\frac{\cancel{240}^{40}}{\cancel{6}_{1}} = \frac{{}^{1}\cancel{6}r}{{}_{1}\cancel{6}}$$
$$40 = r$$

This is actually how we get $r = \frac{d}{t}$ from $d = rt$.

$$d = rt$$
$$\frac{d}{t} = \frac{rt}{t} \qquad \text{divide each side of the equation by } t$$
$$\frac{d}{t} = \frac{r\cancel{t}^{1}}{\cancel{t}_{1}}$$
$$\frac{d}{t} = r$$

Finding Time Traveled

What would you do if you knew the distance traveled and the rate of speed but needed to find the time traveled?

- The formula for distance is $d = rt$. You can find the time traveled by dividing the distance by the rate as follows.

$$\text{time} = \frac{\text{distance}}{\text{rate}}$$

Example of finding time traveled:

If you travel 360 miles at an average rate of 45 miles per hour, your time can be found by dividing the distance, 360, by the rate, 45.

$$t = \frac{d}{r}$$
$$t = \frac{360}{45}$$
$$t = 8$$

Formulas for Distance, Rate or Average Speed, and Time

Review the three formulas below. Each will be used in the following practice.

- total distance is the average rate of speed x total time

  $d = rt$

- average rate of speed is the $\frac{\text{total distance}}{\text{total time}}$

  $r = \frac{d}{t}$

- total time is the $\frac{\text{total distance}}{\text{average rate of speed}}$

  $t = \frac{d}{r}$

Practice

Use the **formula** *below to complete the following table.*

$$d = rt \qquad r = \frac{d}{t} \qquad t = \frac{d}{r}$$

Time, Rate, Distance, and Formula Used

Time (in hours)	Rate (in miles per hour)	Distance (in miles)	Formula Used
2	60		
	65	260	
6		420	
	75	450	
10		550	
	55	495	
7	50		
20		1,100	
	65	975	
5		265	
3	70		
	70	840	
	55	660	
7		455	
24		1,560	
10	62.5		
20	62.5		
	62.5	500	
12		750	