Lesson Three Purpose

- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use concrete and graphic models to derive formulas for finding rate, distance, time, and angle measures. (MA.B.1.4.2)
- Solve real-world problems involving rated measures (miles per hour, feet per second). (MA.B.2.4.2)
- Describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, tables, and graphs. (MA.D.1.4.1)
- Interpret data that has been collected, organized, and displayed in charts, tables, and plots. (MA.E.1.4.1)
- Design and perform real-world statistical experiments, then analyze results and report findings. (MA.E.3.4.1)

Linear Relationships

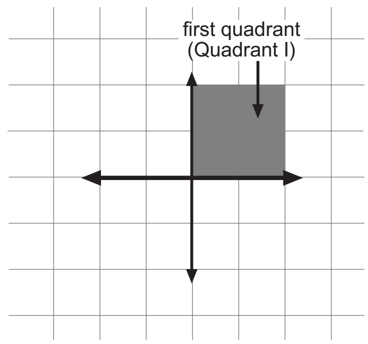
Linear relationships are relationships between **variables** that can be expressed as straight-line graphs. *Linear relationships* are important in mathematics and in everyday life. We will look at why the distance formula, $d = rt$, represents a linear relationship.

To *see* the linear relationship that exists, we will plot data on a **coordinate grid or plane**. First, let's review how to make a graph.

Plans for Making a Graph to Plot Time and Distance

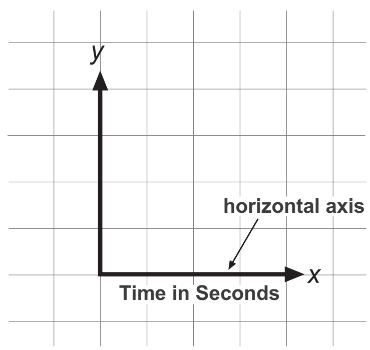
We will begin a graph to plot the distance between when the sound of thunder is heard (time in seconds) and when the lightning strike is seen (distance in feet).

- Time and distance are always **positive numbers**. Only the first **quadrant** or region of the graph is needed.

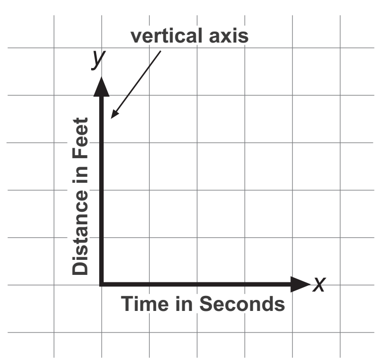


coordinate graph

- Distance *changes* over time, so time is placed on the horizontal (⟷) or ***x*-axis** and labeled "Time in Seconds."

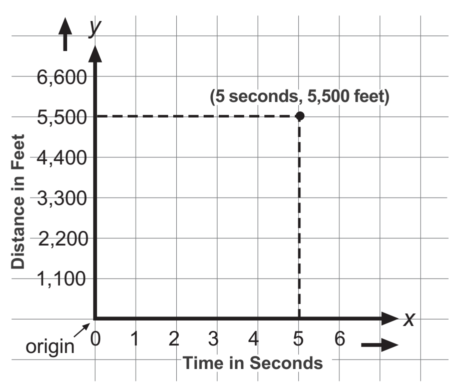


- Distance traveled is placed on the vertical (↕) or ***y*-axis** and labeled "Distance in Feet."



- Every graph needs a title. The title is placed above the graph.
- On this graph, zero (0) is used as the **minimum** value and 6 is the **maximum** value for time. A *scale* or *assigned numeric value* of 1 has been used on the *x*-axis.
- Zero is also the minimum value, and 6,600 is the maximum value for distance. A scale of 1,100 was chosen for the *y*-axis. This is because the rate of the speed of sound is about 1,100 feet per second. However, this is not the only scale that could have been chosen.
- To plot the **ordered pairs** and **graph the points** for the data, start at the **origin** (0, 0). The *origin* is the **intersection** where the *x*-axis and the *y*-axis meet. First, locate the number of seconds on the *x*-axis. Then move from that **point** on the *x*-axis straight up and **parallel (||)** to the *y*-axis. Move to a *point* aligned with the correct distance on the *y*-axis and draw a *point*.

Sound of Thunder and Lightning Strike Distance

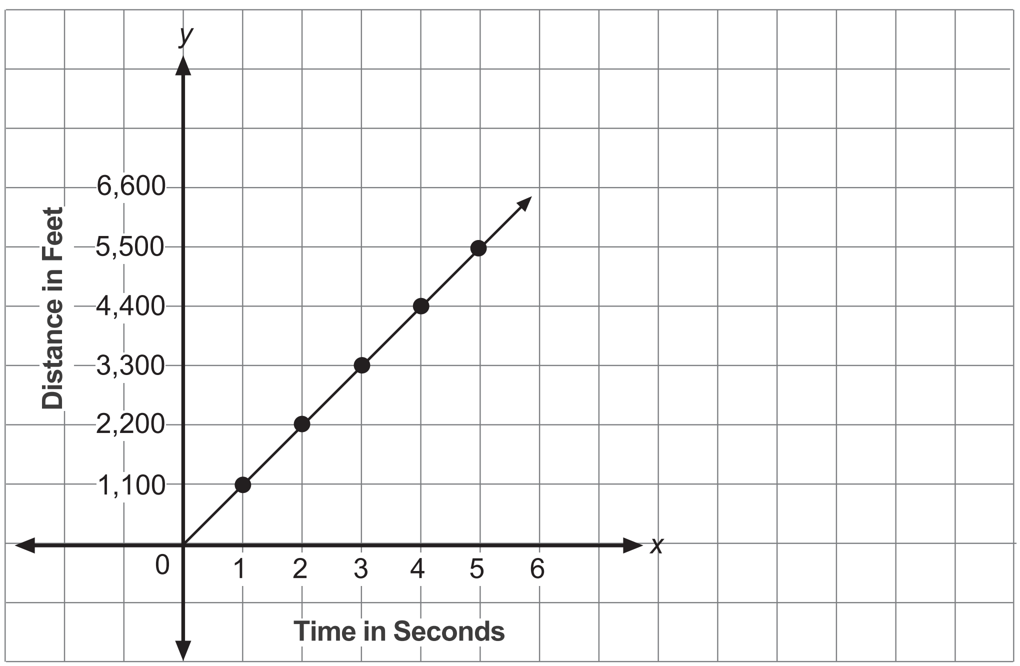


- A set of ordered pairs (5, 5500) has been located on the graph. The 5 is the first number of the ordered pair or the ***x*-coordinate** on the *x*-axis (↔). The 5,500 is the second number of the ordered pair or the ***y*-coordinate**.
- To complete the graph, you would continue to plot the other **coordinates**, or sets of ordered pairs, that correspond to points on the *coordinate plane*.

Sound of Thunder and Lightning Strike Distance

Time in Seconds (*s*)	Distance (*d*) in Feet
0	0
1	1,100
2	2,200
3	3,300
4	4,400
5	5,500

Sound of Thunder and Lightning Strike Distance



Practice

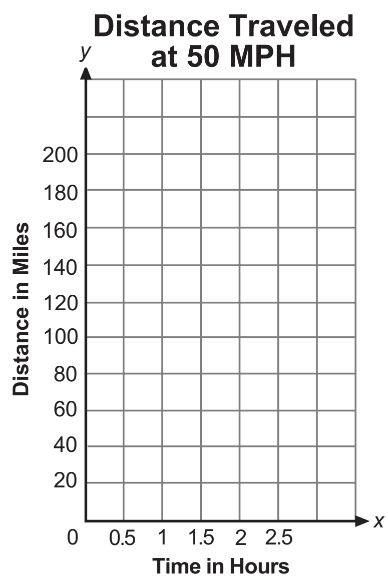
Use the **distance formula** *below to complete the following* **tables** *and* **plot** *the points on the* **coordinate grid** *provided.* **Title** *each graph and* **label** *the* **axes**. **The first graph has been titled and labeled for you.**

distance = rate x time
$d = rt$

1. Distance traveled at a rate of **50 miles per hour (mph)**

Distance Traveled at 50 MPH

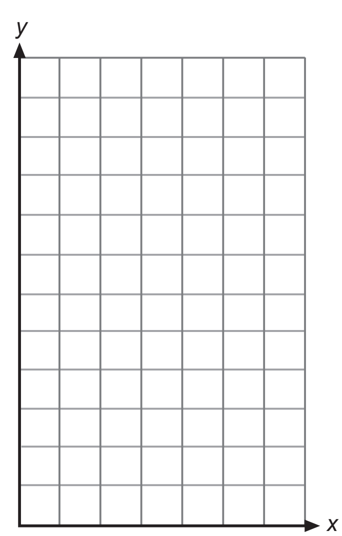
Time in Hours	Distance in Miles
0.5	25
1	
1.5	
2	
2.5	



2. Distance traveled at a rate of **55 mph**

Distance Traveled at 55 MPH

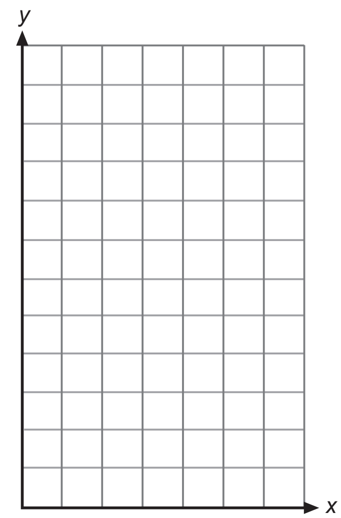
Time in Hours	Distance in Miles
0.5	27.5
1	
1.5	
2	
2.5	



3. Distance traveled at a rate of **60 mph**

Distance Traveled at 60 MPH

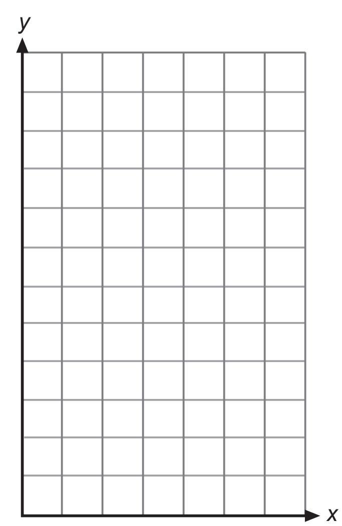
Time in Hours	Distance in Miles
0.5	30
1	
1.5	
2	
2.5	



4. Distance traveled at a rate of **65 mph**

Distance Traveled at 65 MPH

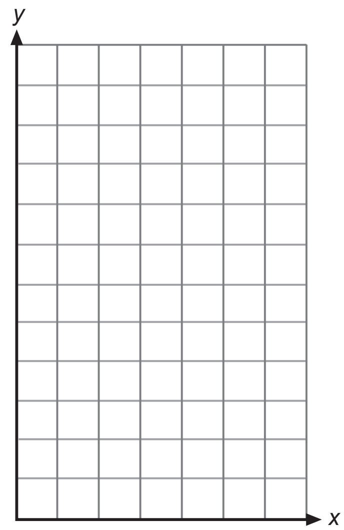
Time in Hours	Distance in Miles
0.5	32.5
1	
1.5	
2	
2.5	



5. Distance traveled at a rate of **70 mph**

Distance Traveled at 70 MPH

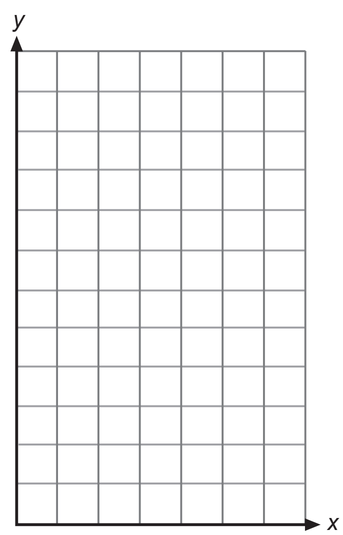
Time in Hours	Distance in Miles
0.5	35
1	
1.5	
2	
2.5	



6. Distance traveled at a rate of **75 mph**

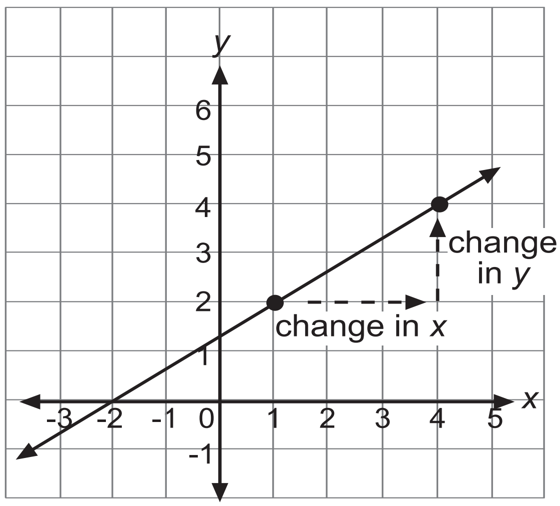
Distance Traveled at 75 MPH

Time in Hours	Distance in Miles
0.5	37.5
1	
1.5	
2	
2.5	



Change of Rate over Time—Slope

We all know that some mountains are steeper than others. Lines in a **coordinate plane** also have steepness. In math, the steepness of a line is called its **slope**. *Slope* can be used to describe a **rate of change**. The *rate of change* tells how a quantity is changing over time. The *vertical* (↕) change is called the *change in y* and the *horizontal* (↔) change is called the *change in x*. On the coordinate grid below, the line slopes upward from left to right, indicating a positive slope.



slope = rate of change

Practice

A constant **rate of change** *can be seen in each* **table** *from the previous practice numbers 1-6. Use the* **answers from the previous practice** *to complete the following.*

1. In the table for **50 mph**, as the time increases by ____________ hour,

   the distance increases by ____________ miles.

2. In the table for **55 mph**, as the time increases by ____________ hour,

   the distance increases by ____________ miles.

3. In the table for **60 mph**, as the time increases by ____________ hour,

   the distance increases by ____________ miles.

4. In the table for **65 mph**, as the time increases by ____________ hour,

   the distance increases by ____________ miles.

5. In the table for **70 mph**, as the time increases by ____________ hour,

   the distance increases by ____________ miles.

6. In the table for **75 mph**, as the time increases by ____________ hour,

   the distance increases by ____________ miles.

Answer the following using the **graphs** *from the previous practice.*

7. A constant rate of change can be seen in each of the graphs because the points for each graph lie in a straight ____________ .

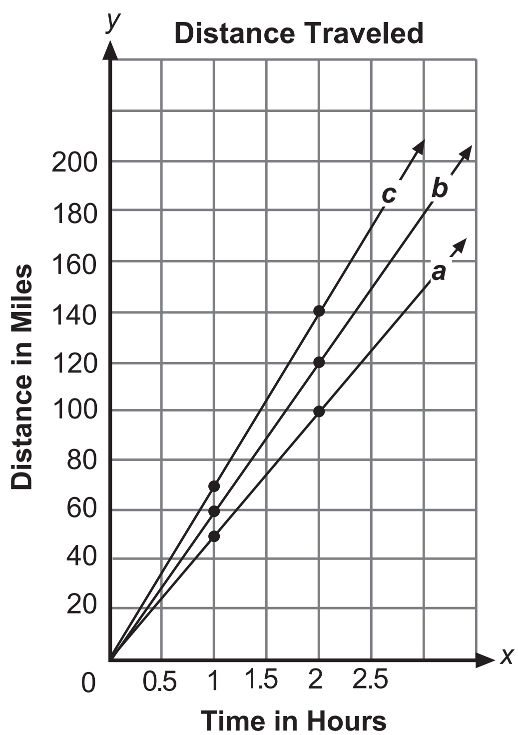
Complete the following.

8. The rate of change is constant in linear relationships. The following graph has three lines and each is labeled either *a*, *b*, or *c*.

   Line *a* represents the distance traveled at ____________ mph.

   Line *b* represents the distance traveled at ____________ mph.

   Line *c* represents the distance traveled at ____________ mph.



9. Lines *a*, *b*, and *c* on the graph provided in question 8 are *not* **parallel lines** because their slopes are different. The rate of change controls the slope of a line. Look carefully at the lines.

   Line ____________ has the steepest slope and represents a rate of

   change of ____________ mph.

   Line ____________ has the gentlest slope and represents a rate of

   change of ____________ mph.

10. If a line were added to the graph representing distance traveled at 65

    mph, it would lie between lines ____________ and ____________ .

11. If a line were added to the graph representing a distance traveled at

    40 mph, its slope would be ____________ (greater or less) than the

    slopes of lines *a*, *b*, and *c* and it would lie (above or below)

    ____________ the line on the graph.

12. Using a graphing calculator or computer software, produce the tables and graphs from the previous practice for numbers 1-6.

    Note: When a general equation, such as $y = 60x$, is entered to determine the distance traveled (y) at 60 mph for any number (x) miles, a table of values and a graph can be produced. Graphing calculators can have different keyboards and displays. Follow your teacher's instruction.

    ____

Practice

Use the list below to write the correct term for each definition on the line provided.

______	1. numbers that correspond to points on a coordinate plane in the form (x, y)	A.	coordinate grid or plane
______	2. being an equal distance at every point so as to never intersect	B.	coordinates
______	3. the smallest amount or number allowed or possible	C.	graph of a point
______	4. the point of intersection of the x- and y-axes in a rectangular coordinate system, where the x-coordinate and y-coordinate are both zero (0)	D.	maximum
______	5. a two-dimensional network of horizontal and vertical lines that are parallel and evenly-spaced	E.	minimum
______	6. the vertical number line on a rectangular coordinate system	F.	origin
______	7. the horizontal number line on a rectangular coordinate system	G.	parallel (||)
______	8. the point assigned to an ordered pair on a coordinate plane	H.	x-axis
______	9. the largest amount or number allowed or possible	I.	y-axis

Practice

Match each definition with the correct term. Write the letter on the line provided.

coordinate plane	**positive numbers**	**slope**
intersection	**quadrant**	***x*-coordinate**
ordered pair	**rate of change**	***y*-coordinate**
parallel lines		

____________________ 1. the first number of an ordered pair

____________________ 2. the second number of an ordered pair

____________________ 3. the point at which two lines or curves meet

____________________ 4. the plane containing the x- and y-axes

____________________ 5. numbers greater than zero

____________________ 6. the ratio of change in the vertical axis (y-axis) to each unit change in the horizontal axis (x-axis) in the form $\frac{\text{rise}}{\text{run}}$ or $\frac{\Delta y}{\Delta x}$

____________________ 7. two lines in the same plane that are a constant distance apart; lines with equal slopes

____________________ 8. any of four regions formed by the axes in a rectangular coordinate system

____________________ 9. the location of a single point on a rectangular coordinate system where the first and second values represent the position relative to the x-axis and y-axis, respectively

____________________ 10. how a quantity is changing over time