Lesson Two Purpose

- Associate verbal names, written word names, and standard numerals with integers, rational numbers, and real numbers. (MA.A.1.4.1)
- Understand the relative size of integers, rational numbers, and real numbers. (MA.A.1.4.2)
- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand and use the real number system. (MA.A.2.4.2)
- Understand and explain the effects of addition, subtraction, multiplication, and division on real numbers, including square roots, exponents, and appropriate inverse relationships. (MA.A.3.4.1)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)
- Solve real-world problems involving rated measures (miles per hour, feet per second). (MA.B.2.4.2)
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)
- Represent real-world problem situations using finite graphs. (MA.D.2.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)

Subtraction of Integers

Looking for Patterns

Study the problems below. Look for patterns. Remember, patterns are predictable.

$8 - 4 = 4$

$8 - 3 = 5$

$8 - 2 = 6$

$8 - 1 = 7$

$8 - 0 = 8$

$8 - (-1) = 9$

$8 - (-2) = 10$

$8 - (-3) = 11$

$8 - (-4) = 12$

Think about This!

From the pattern above, we can say the following:

- As the number being subtracted from 8 **decreases**, the **difference increases**.

  Example: When 4 is subtracted from 8, the difference is 4.
  $8 - 4 = 4$

  When -4 is subtracted from 8, the difference is 12.
  $8 - (-4) = 12$

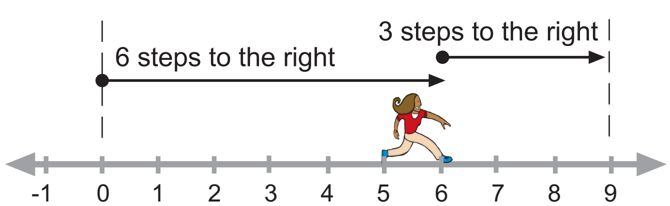
This is difficult for some people to grasp. Perhaps modeling subtraction will help clarify this. *Subtracting* a number from 8 and *adding* the opposite of that number to 8 will yield the same result.

8 – 4 = 4	and	8 + (-4) = 4
8 – 1 = 7	and	8 + (-1) = 7
8 – (-2) = 10	and	8 + 2 = 10

Using the Number Line Method to Model Adding and Subtracting Integers

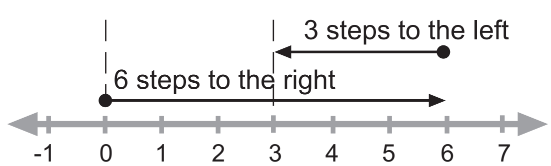
When you were first introduced to addition and subtraction of whole numbers, you may have used a number line drawn on the floor or on paper.

When adding 6 + 3, you likely started at zero, took 6 steps to the right and then another 3 steps to the right. You wound up on 9.



6 + 3 = 9

When subtracting 3 from 6, you likely started at zero, took 6 steps to the right, and then 3 steps to the left, winding up on 3.



6 – 3 = 3

In Lesson One of this unit, you moved to the *right* when *adding a positive number* and to the *left* when *adding a negative number*.

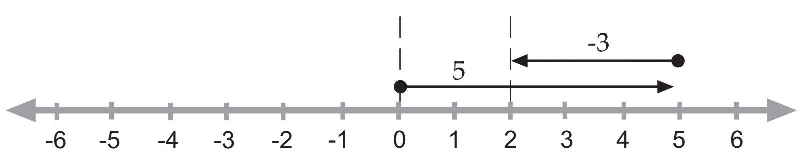
When using a number line for subtraction of integers, we always begin at zero and move the appropriate number of steps to the left or right to represent the first number (sometimes called the *minuend*). The minuend is the number you subtract from. The *subtrahend* is the number being subtracted.

6	← minuend
– 3	← subtrahend
3	← difference

- If the number to be subtracted is a *positive* number, we move to the *left* as we have in the past.
- If the number to be subtracted is a *negative* number, we move to the *right*, which is the opposite of what we have done in the past.

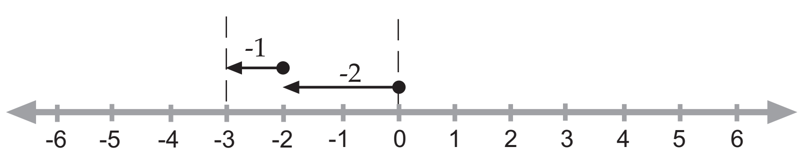
Number Line Models

5 – 3 = 2



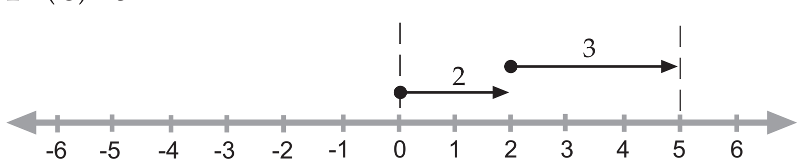
5 – 3 = 2

-2 – 1 = -3



-2 – 1 = -3

2 – (-3) = 5



2 – (-3) = 5

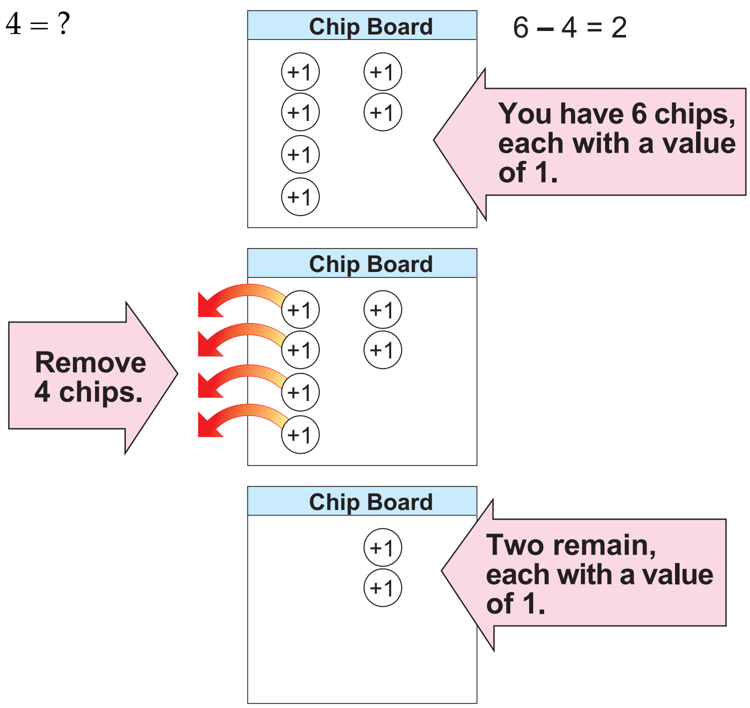
Using the Chip Method to Model Adding and Subtracting Integers

In Lesson One of this unit, we paired the positive 1s with the negative 1s to make zero until no pairs were left when adding. We then counted the chips that remained.

When using the chip method to model for subtraction we "take away."

Example 1:

$6 - 4 = ?$

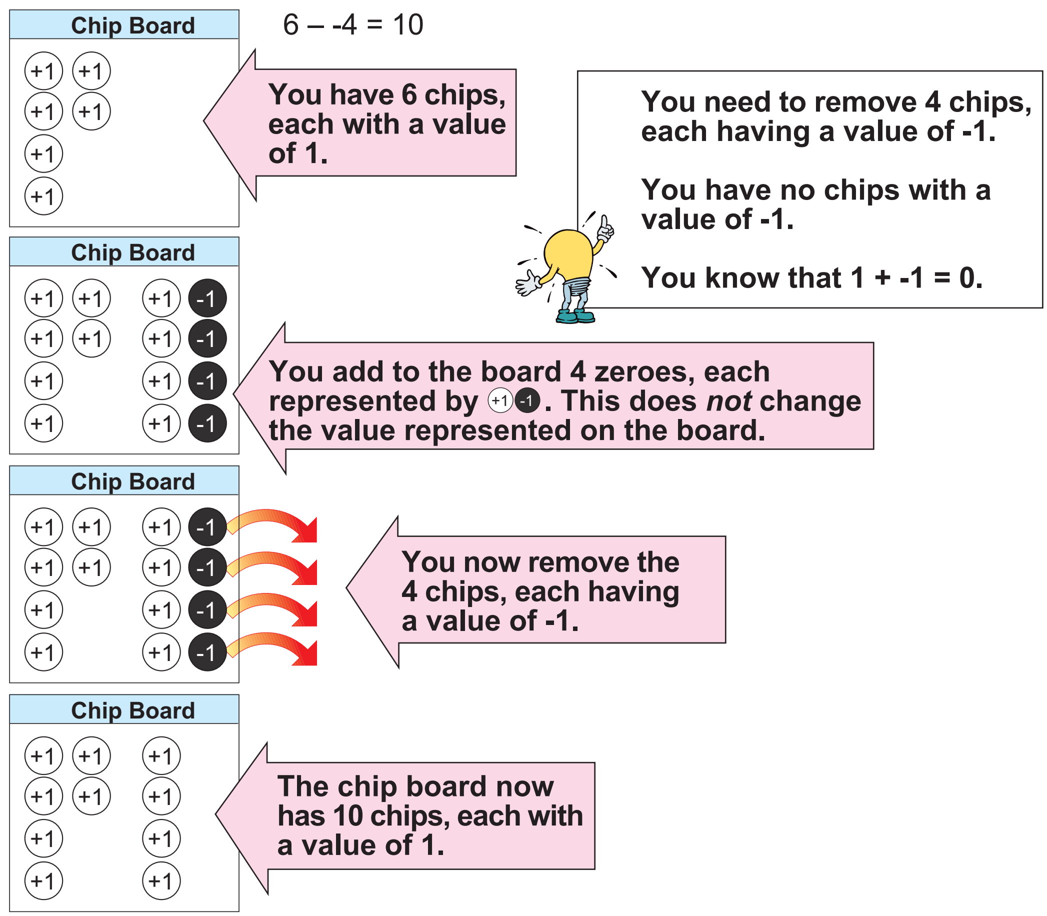


- We begin with 6 positive chips and take 4 of them away,
- leaving 2 positive chips.

So $6 - 4 = 2$.

Example 2:

6 – (-4) = ?

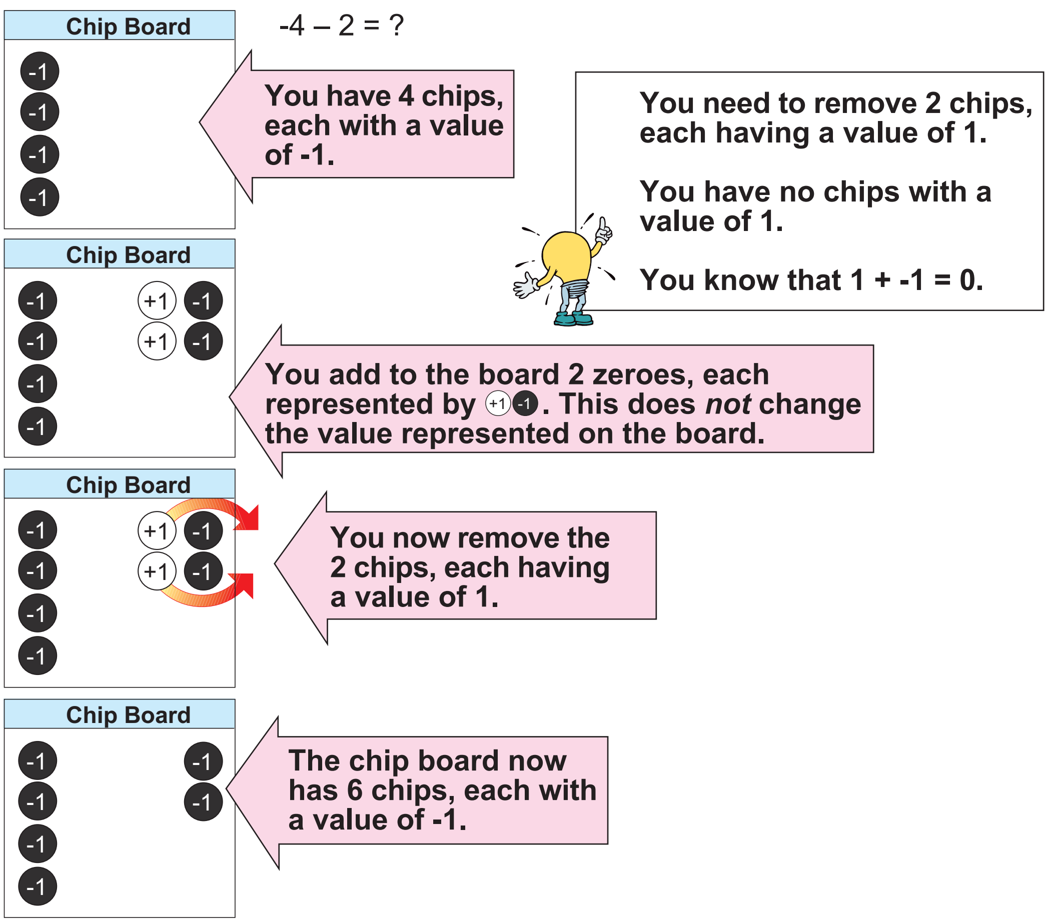


- We would begin with 6 positive chips.
- There are no negative chips and we need to take 4 negative chips away.
- We therefore place 4 positive and 4 negative chips on the board. This is like adding zero.
- We now have 4 negative chips to take away, leaving 10 positive chips on the board.

So 6 – (-4) = 10.

Example 3:

-4 – 2 = ?



- We begin with 4 negative chips.
- There are no positive chips and we need to take 2 positive chips away.
- We therefore place 2 positive chips and 2 negative chips on the board. This is like adding zero.
- We now have 2 positive chips to take away, leaving 6 negative chips on the board.

So -4 – 2 = -6.

Review: Subtracting Integers

Definition of Subtraction
***a* – *b* = *a* + (-*b*)**

Examples: 8 – 10 = 8 + (-10) = -2

12 – 20 = 12 + (-20) = -8

-2 – 3 = -2 + (-3)= -5

Even if we have 8 – (-8), this becomes

8 plus the opposite of -8, which equals 8.

8 + -(-8) =

8 + 8 = 16

Shortcut Two negatives become one positive!

10 – (-3) becomes 10 plus 3.

10 + 3 = 13

And -10 – (-3) becomes -10 plus 3.

-10 + 3 = -7

Generalization: Subtracting Integers

Subtracting an integer is the same as adding its opposite.

a – *b* = *a* + (-*b*)

Rules to Subtract Integers

Subtract	Outcome
positive – positive	The difference is *positive* if the first number is greater—however, if the second number is greater, it is *negative*.
negative – positive	The difference is *negative*.
positive – negative	The difference is *positive*.
negative – negative	The difference is *negative* if the first absolute value is greater—however, if the second absolute value is greater, it is *positive*.

Practice

Solve the following.

1. $6 - 4 =$

2. $5 - (-3) =$

3. $-14 - 5 =$

4. $-12 - (-2) =$

5. $-57 - 3 =$

6. $18 - 24 =$

7. $21 - (-3) =$

8. $26 - (-26) =$

9. $-37 - 17 =$

10. $-37 - (-17) =$

11. *Complete the following statements.*

a. When a *positive* integer is subtracted from a *positive* integer, the result is ____________ (always, sometimes, never) positive.

Hint: See numbers 1 and 6.

b. When a *negative* integer is subtracted from a *negative* integer, the result is ____________ (always, sometimes, never) positive.

Hint: See numbers 4 and 10.

c. When a *negative* integer is subtracted from a *positive* integer, the result is ____________ (always, sometimes, never) positive.

Hint: See numbers 2, 7, and 8.

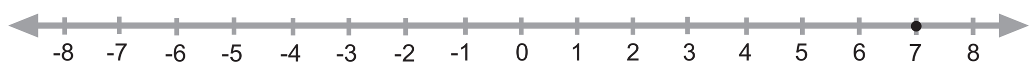
d. When a *positive* integer is subtracted from a *negative* integer, the result is ____________ (always, sometimes, never) positive.

Hint: See numbers 3, 5, and 9.

Answer the following.

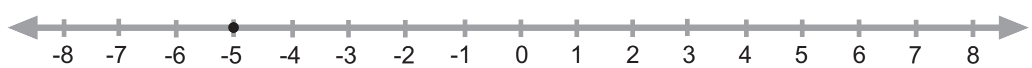
12. The distance between an unknown number and 7 on a number line is 9 units. Give two possibilities for the unknown number.

    Answer: ____________ ____________



13. The distance between an unknown number and -5 on a number line is 12 units. Give two possibilities for the unknown number.

    Answer: ____________ ____________



14. Explore adding and subtracting with a scientific calculator. Consider the difference in the key used for subtraction on your calculator and the key used to indicate that a number being entered is a negative number.

Continuing to Keep Equations in Balance

Think about This!

Let's consider two problems similar to the ones visited in Lesson One of this unit.

Margaret earns $4 *more* per hour than Susie. If Margaret earns $20 per hour, what is Susie's hourly rate?

In the equation,

$$s + 4 = 20,$$

s represents Susie's hourly rate.

We might think:

- "To what number can I add 4 and the result be 20?"

$$? + 4 = 20$$

$$16 + 4 = 20$$

Susie's hourly rate is $16.

However, you might also choose to use the *inverse operation of addition*, which is *subtraction*.

$$s + 4 = 20$$

$$s + 4 \mathbf{- 4} = 20 \mathbf{- 4}$$

$$s = 16$$

← subtract 4 from both sides →

$$\begin{array}{rcr} s + \cancel{4} & = & 20 \\ -\cancel{4} & & -4 \\ \hline s & = & 16 \end{array}$$

Susie's hourly rate is $16.

Or, you might also choose to *define subtraction as adding the opposite* and solve the problem this way:

$$s + 4 = 20$$

$$s + 4 + \mathbf{(-4)} = 20 + \mathbf{(-4)}$$

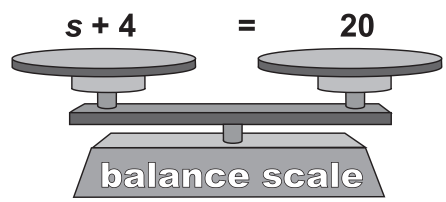
$$s = 16$$

add -4 to both sides

$$\begin{array}{rcr} s + \cancel{4} & = & 20 \\ \underline{+ \cancel{-4}} & & \underline{+ -4} \\ s & = & 16 \end{array}$$

The value on the *left* side of the equation, $s + 4$, is the same as the value on the *right* side of the equation, 20. If I add -4 to the value on the *left* side, I must add -4 to the value on the *right* side if the equation is to remain true. This property is called the *addition property of equality*.

$s + 4 = 20$



$s + 4 + (-4) = 20 + (-4)$
$s = 16$

keeping an equation balanced

Remember: Equality properties help you keep an equation *balanced*.

Jack and Tom played a game in which Tom's score was 2 more than Jack's. Tom's score was -5. What was Jack's score?

In the equation, j represents Jack's score.

$$j + 2 = -5$$

$$j + 2 \mathbf{- 2} = -5 \mathbf{- 2}$$

$$j = -7$$

subtract 2 from both sides

$$\begin{array}{rcr} j + \cancel{2} & = & -5 \\ \underline{- \cancel{2}} & & \underline{- 2} \\ j & = & -7 \end{array}$$

or

$$j + 2 = -5$$

$$j + 2 + \mathbf{(-2)} = -5 + \mathbf{(-2)}$$

$$j = -7$$

add -2 to both sides

$$\begin{array}{rcr} j + \cancel{2} & = & -5 \\ \underline{+ \cancel{-2}} & & \underline{+ -2} \\ j & = & -7 \end{array}$$

Jack's score was -7.

Practice

Solve the following.

1. $a + 25 = 40$
2. $b + 25 = 15$
3. $c + 13 = 30$
4. $d + 13 = 5$
5. $e + 12 = -2$
6. $f + 14 = -20$
7. $g + 5 = -8$
8. $h + 60 = 24$