



## Lesson Three Purpose

- Associate verbal names, written word names, and standard numerals with integers, rational numbers, and real numbers. (MA.A.1.4.1)
- Understand the relative size of integers, rational numbers, and real numbers. (MA.A.1.4.2)
- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand and use the real number system. (MA.A.2.4.2)
- Understand and explain the effects of addition, subtraction, multiplication, and division on real numbers, including square roots, exponents, and appropriate inverse relationships. (MA.A.3.4.1)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)
- Solve real-world problems involving rated measures (miles per hour, feet per second). (MA.B.2.4.2)
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)
- Represent real-world problem situations using finite graphs. (MA.D.2.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)



## Multiplication of Integers

### Looking for Patterns

Study the problems below for patterns.

$$4 \times 3 = 12 \qquad -4 \times 3 = -12$$

$$4 \times 2 = 8 \qquad -4 \times 2 = -8$$

$$4 \times 1 = 4 \qquad -4 \times 1 = -4$$

$$4 \times 0 = 0 \qquad -4 \times 0 = 0$$

$$4 \times -1 = -4 \qquad -4 \times -1 = 4$$

$$4 \times -2 = -8 \qquad -4 \times -2 = 8$$

$$4 \times -3 = -12 \qquad -4 \times -3 = 12$$

### Think about This!

1. As 4 is multiplied by a **factor** 1 *less than* the previous factor, the **product** is 4 *less than* the previous product.



**Remember:** A *factor* is a number that divides evenly into another number.

$$4 \times 3 = 12$$

$$4 \times 2 = 8$$

2. As -4 is multiplied by a factor 1 *less than* the previous factor, the product is 4 *greater than* the previous product.

$$-4 \times 3 = -12$$

$$-4 \times 2 = -8$$

3. The product of a number and zero is *zero*.

$$4 \times 0 = 0$$

$$-4 \times 0 = 0$$



4. The product of a number and 1 is that *same number*.

$$4 \times 1 = 4$$

5. The product of two positive numbers, such as 4 and 3, is a *positive* number.

$$4 \times 3 = 12$$

6. The product of two negative numbers, such as -4 and -2, is a *positive* number.

$$-4 \times -2 = 8$$

7. The product of one negative number and one positive number, such as -4 x 2 or 4 x -3, is a *negative* number.

$$-4 \times 2 = -8$$

$$4 \times -3 = -12$$

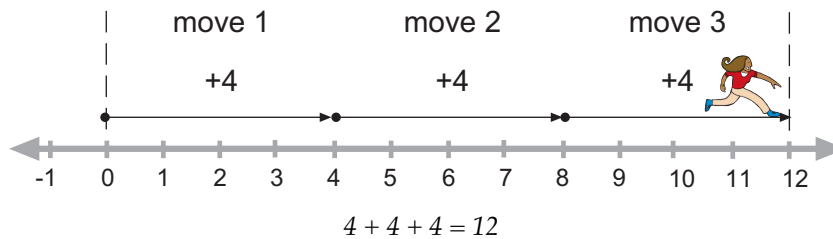
You can see that the sign of a *product* depends on the signs of the numbers being multiplied. Therefore, you can use the following rules for multiplying integers.

#### Rules to Multiply Integers

- The product of two positive integers is *positive*.
- The product of two negative integers is *positive*.
- The product of two integers with different signs is *negative*.
- The product of any integer and 0 is 0.



Let's consider using the number line to model multiplication.



To show  $3 \times 4 = 12$  on the number line, we will show 3 moves of 4 units each. We would do the same for the problem  $4 + 4 + 4 = 12$ , since we can think of multiplication as repeated addition.



## Practice

Use the method of your choice to solve the following.

1.  $5 \times 6 =$

5.  $3 \times (-18) =$

2.  $6 \times (-7) =$

6.  $2 \times 4 =$

3.  $-4 \times 8 =$

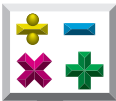
7.  $-20 \times (-20) =$

4.  $-5 \times (-20) =$

8.  $-6 \times (-6) =$

9. The temperature was 83 degrees at 9:00 PM and dropped an average of 1.5 degrees per hour for the next 9 hours. What was the temperature at 6:00 AM?

*Answer:* \_\_\_\_\_ degrees



## Dividing Integers

Division is the *inverse* of multiplication. The same patterns we saw in multiplication will exist in division.

We know that

$$24 \div 3 = 8 \quad \leftarrow \text{The **quotient** of two positive integers is } \textit{positive}.$$

$$\text{because } 8 \times 3 = 24. \quad \leftarrow \text{The product of two positive integers is } \textit{positive}.$$

The same will be true when we are dividing integers.

$$24 \div (-3) = -8 \quad \leftarrow \text{The } \textit{quotient} \text{ of two integers with different signs is } \textit{negative}.$$

$$\text{because } -8 \times (-3) = 24 \quad \leftarrow \text{The product of two negative integers is } \textit{positive}.$$

and

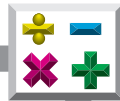
$$-24 \div 3 = -8 \quad \leftarrow \text{The quotient of two integers with different signs is } \textit{negative}.$$

$$\text{because } -8 \times 3 = -24 \quad \leftarrow \text{The product of two integers with different signs is } \textit{negative}.$$

and

$$-24 \div (-3) = 8 \quad \leftarrow \text{The quotient of two negative integers is } \textit{positive}.$$

$$\text{because } 8 \times -3 = -24 \quad \leftarrow \text{The product of two integers with different signs is } \textit{negative}.$$



In summary,

**same signs → positive  
quotient (or product)**

$$24 \div 3 = 8 \quad \text{because } 8 \times 3 = 24$$

$$-24 \div (-3) = 8 \quad \text{because } 8 \times (-3) = -24$$

**different signs → negative  
quotient (or product)**

$$-24 \div 3 = -8 \quad \text{because } -8 \times 3 = -24$$

$$24 \div (-3) = -8 \quad \text{because } -8 \times -3 = 24$$

Below are the rules used to divide integers.

#### Rules to Divide Integers

- The quotient of two positive integers is *positive*.
- The quotient of two negative integers is *positive*.
- The quotient of two integers with different signs is *negative*.
- The quotient of 0 divided by any nonzero integer is *0*.



## Practice

Complete the following.

1.  $35 \div 5 =$

6.  $-400 \div 25 =$

2.  $49 \div (-7) =$

7.  $-625 \div (-25) =$

3.  $225 \div (-15) =$

8.  $1,000 \div (-10) =$

4.  $-121 \div 11 =$

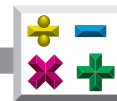
9.  $-1,000 \div 100 =$

5.  $169 \div (-13) =$

10.  $-10,000 \div (-100) =$

11. The temperature of 69 degrees dropped to 44 degrees at an average rate of 6.25 degrees per hour. How many hours did the total drop of 25 degrees require?

Answer: \_\_\_\_\_ hours



## Solving Word Problems

Consider the following problem:

Bob's hourly wage is 3 times as much as Jerry's. If Bob earns \$33.00 per hour, what is Jerry's hourly wage?

If  $j$  represents Jerry's hourly wage then

$$3j = 33$$

- You might think:

Three times some number is 33.

$$3 \times ? = 33$$

That number is 11.

- You might think:

The **multiplicative inverse (reciprocal)** of 3 is  $\frac{1}{3}$  because  $\frac{1}{3}$  times 3 is 1.

$$\frac{1}{3} \times 3 = 1$$

I'll multiply both sides of the equation by  $\frac{1}{3}$ .

$$\begin{array}{ccc} 3j = 33 & & \\ \frac{1}{3}(3j) = \frac{1}{3}(33) & \xleftarrow{\text{multiply both}} & \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}j}{1} = \frac{1}{\cancel{3}} \cdot \frac{33}{1} \\ j = 11 & \xrightarrow{\text{sides by } \frac{1}{3}} & j = 11 \end{array}$$



- You might think:

Division is the inverse operation for multiplication.  
When 3 is divided by 3, the quotient is 1.

$$3 \div 3 = 1$$

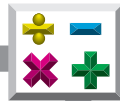
I'll divide both sides of the equation by 3.

$$3j = 33$$

$$(3j) \div 3 = 33 \div 3 \quad \leftarrow \text{divide both sides by 3}$$

$$j = 11$$

Jerry's hourly wage is \$11.00.



## Practice

Use the method of your choice to solve the following.

1.  $4a = 32$

5.  $-17e = -289$

2.  $-2b = 46$

6.  $20f = 300$

3.  $25c = -400$

7.  $14g = -42$

4.  $15d = 225$

8.  $-24h = -96$



## Practice

Use the list below to make each statement **true**. One or more numbers will be used more than once.

-6	-4	-2	2	4	6
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1. \_\_\_\_\_ + \_\_\_\_\_ = -8

and

\_\_\_\_\_ + \_\_\_\_\_ = -8

2. \_\_\_\_\_ + \_\_\_\_\_ = 0

and

\_\_\_\_\_ + \_\_\_\_\_ = 0

and

\_\_\_\_\_ + \_\_\_\_\_ = 0

3. \_\_\_\_\_ + \_\_\_\_\_ = -2

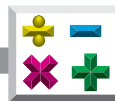
and

\_\_\_\_\_ + \_\_\_\_\_ = -2

4. \_\_\_\_\_ + \_\_\_\_\_ = 2

and

\_\_\_\_\_ + \_\_\_\_\_ = 2



## Practice

Use the list below to make each statement **true**. One or more numbers will be used more than once.

-5	-3	-2	2	3	5
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1. \_\_\_\_\_ x \_\_\_\_\_ = 4

and

\_\_\_\_\_ x \_\_\_\_\_ = 4

2. \_\_\_\_\_ x \_\_\_\_\_ = -4

3. \_\_\_\_\_ x \_\_\_\_\_ = 25

and

\_\_\_\_\_ x \_\_\_\_\_ = 25

4. \_\_\_\_\_ x \_\_\_\_\_ = -15

and

\_\_\_\_\_ x \_\_\_\_\_ = -15

5. \_\_\_\_\_ x \_\_\_\_\_ = 9

and

\_\_\_\_\_ x \_\_\_\_\_ = 9



6. \_\_\_\_\_ x \_\_\_\_\_ = -25

7. \_\_\_\_\_ x \_\_\_\_\_ = -9

8. \_\_\_\_\_ x \_\_\_\_\_ = 6

and

\_\_\_\_\_ x \_\_\_\_\_ = 6

9. \_\_\_\_\_ x \_\_\_\_\_ = -6

and

\_\_\_\_\_ x \_\_\_\_\_ = -6



## Practice

Use the list below to write the correct term for each definition on the line provided.

<b>difference</b>	<b>natural numbers</b>	<b>product</b>
<b>factor</b>	<b>negative numbers</b>	<b>quotient</b>
<b>integers</b>	<b>positive numbers</b>	<b>sum</b>
<b>multiplicative inverse</b>		

- \_\_\_\_\_ 1. the numbers in the set  $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$
- \_\_\_\_\_ 2. the numbers in the set  $\{1, 2, 3, 4, 5, \dots\}$
- \_\_\_\_\_ 3. the result of adding numbers together
- \_\_\_\_\_ 4. the result of dividing two numbers
- \_\_\_\_\_ 5. numbers greater than zero
- \_\_\_\_\_ 6. numbers less than zero
- \_\_\_\_\_ 7. a number or expression that divides evenly into another number
- \_\_\_\_\_ 8. any two numbers with a product of 1
- \_\_\_\_\_ 9. a number that is the result of subtraction
- \_\_\_\_\_ 10. the result of multiplying numbers together

