Unit 3: Examining Formulas and Relationships with Geometric Shapes

This unit emphasizes using concrete and graphic models and algebraic knowledge to find the perimeter of various geometric shapes.

Unit Focus

Number Sense, Concepts, and Operations

- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand and use the real number system. (MA.A.2.4.2)
- Select and justify alternative strategies, such as using properties of numbers, including inverse, identity, distributive, associative, and transitive, that allow operational shortcuts for computational procedures in real-world or mathematical problems. (MA.A.3.4.2)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)

Measurement

- Use concrete and graphic models to derive formulas for finding perimeter, area, surface area, circumference, and volume of two- and three-dimensional shapes, including rectangular solids, cylinders, cones, and pyramids. (MA.B.1.4.1)

- Relate the concepts of measurement to similarity and proportionality in real-world situations. (MA.B.1.4.3)
- Solve real-world problems involving rated measures (miles per hour, feet per second). (MA.B.2.4.2)
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)

Geometry and Spatial Sense

- Represent and apply geometric properties and relationships to solve real-world and mathematical problems including ratio and proportion. (MA.C.3.4.1)

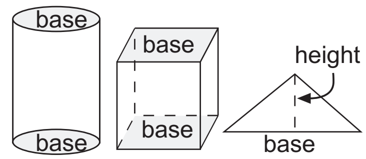
Algebraic Thinking

- Describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, tables, and graphs. (MA.D.1.4.1)
- Represent real-world problem situations using finite graphs. (MA.D.2.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)

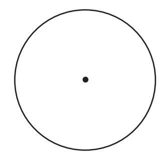
Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

base (b) (geometric) .... the line or plane of a geometric figure, from which an altitude can be constructed, upon which a figure is thought to rest

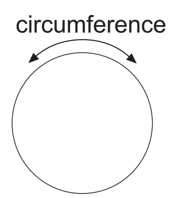


center (of a circle) .... the point from which all points on the circle are the same distance



circle .... the set of all points in a plane that are all the same distance from a given point called the center

circumference (C) .... the distance around a circle



coefficient .... the number part in front of an algebraic term signifying multiplication
Example: In the expression $8x^2 + 3xy - x$,

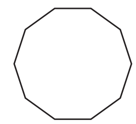
- the coefficient of x^2 is 8 (because $8x^2$ means $8 \bullet x^2$)
- the coefficient of xy is 3 (because $3xy$ means $3 \bullet xy$)
- the coefficient of $-x$ is 1 (because $-1 \bullet x = -x$).

In general algebraic expressions, coefficients are represented by letters that may stand for numbers. In the expression $ax^2 + bx + c = 0$, a, b, and c are coefficients, which can take any number.

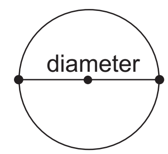
commutative property .... the order in which two numbers are added or multiplied does *not* change their sum or product
Example: 2 + 3 = 3 + 2 *or* 4 x 7 = 7 x 4

congruent (≅) .... figures or objects that are the same shape and size

decagon .... a polygon with 10 sides

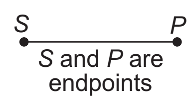


diameter (*d*) .... a line segment from any point on the circle passing through the center to another point on the circle



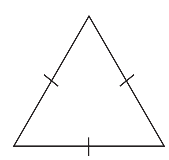
distributive property .... the product of a number and the sum or difference of two numbers is equal to the sum or difference of the two products
Example: $x(a + b) = ax + bx$

endpoint .... either of two points marking the end of a line segment



equation .... a mathematical sentence in which two expressions are connected by an equality symbol
Example: $2x = 10$

equilateral triangle .... a triangle with three congruent sides

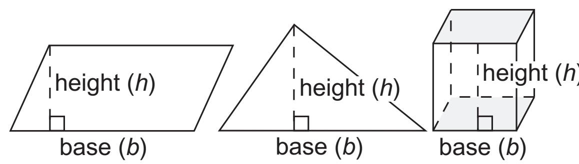


exponent (exponential form) ...... the number of times the base occurs as a factor
Example: 2^3 is the exponential form of 2 x 2 x 2. The numeral two (2) is called the *base*, and the numeral three (3) is called the *exponent*.

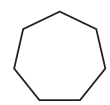
expression .... a collection of numbers, symbols, and/or operation signs that stands for a number
Example: $4r^2$; $3x + 2y$; $\sqrt{25}$
Expressions do *not* contain equality (=) or inequality (<, >, ≤, ≥, or ≠) symbols.

formula .... a way of expressing a relationship using variables or symbols that represent numbers

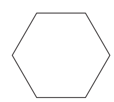
height (*h*) .... a line segment extending from the vertex or *apex* (highest point) of a figure to its base and forming a right angle with the base or plane that contains the base



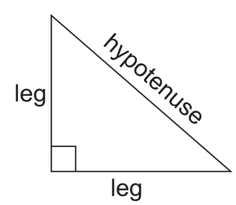
heptagon .... a polygon with seven sides



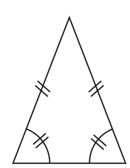
hexagon .... a polygon with six sides



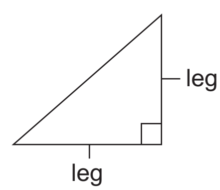
hypotenuse .... the longest side of a right triangle; the side opposite the right angle



isosceles triangle .... a triangle with two congruent sides and two congruent angles



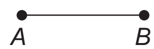
leg .... in a right triangle, one of the two sides that form the right angle



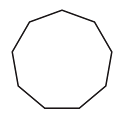
length (*l*) .... a one-dimensional measure that is the measurable property of line segments

like terms .... terms that have the same variables and the same corresponding exponents
Example: In $5x^2 + 3x^2 + 6$, $5x^2$ and $3x^2$ are like terms

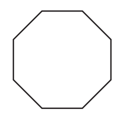
line segment (—) .... a portion of a line that consists of two defined endpoints and all the points in between



Example: The line segment *AB* is between point *A* and point *B* and includes point *A* and point *B*.



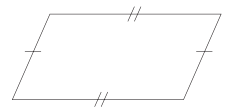
nonagon .... a polygon with nine sides

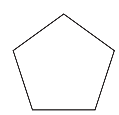


octagon .... a polygon with eight sides

parallel (||) .... being an equal distance at every point so as to never intersect

parallelogram .... a quadrilateral with two pairs of parallel sides





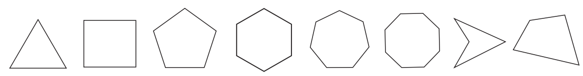
pentagon .... a polygon with five sides

perimeter (*P*) .... the distance around a polygon

pi (π) .... the symbol designating the ratio of the circumference of a circle to its diameter; an irrational number with common approximations of either 3.14 or $\frac{22}{7}$

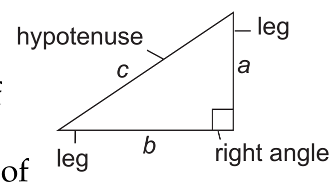
point .... a specific location in space that has no discernable length or width

polygon .... a closed-plane figure, having at least three sides that are line segments and are connected at their endpoints
Example: triangle (3 sides),
quadrilateral (4 sides),
pentagon (5 sides), hexagon (6 sides),
heptagon (7 sides), octagon (8 sides);
concave, convex

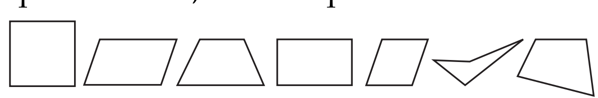


product .... the result of multiplying numbers together
Example: In 6 x 8 = 48,
48 is the product.

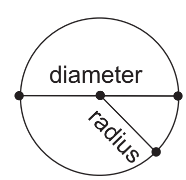
Pythagorean theorem .... the square of the hypotenuse (*c*) of a right triangle is equal to the sum of the square of the legs (*a* and *b*), as shown in the equation $c^2 = a^2 + b^2$



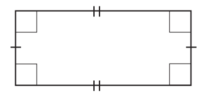
quadrilateral .... polygon with four sides
Example: square, parallelogram, trapezoid, rectangle, rhombus, concave quadrilateral, convex quadrilateral



radius (*r*) .... a line segment extending from the center of a circle or sphere to a point on the circle or sphere; (plural: *radii)*

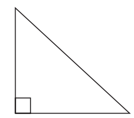


rectangle .... a parallelogram with four right angles



regular polygon .... a polygon that is both *equilateral* (all sides congruent) and *equiangular* (all angles congruent)

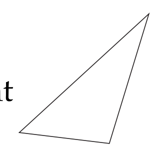
right triangle .... a triangle with one right angle



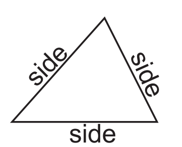
rounded number .... a number approximated to a specified place
Example: A commonly used rule to round a number is as follows.

- If the digit in the first place after the specified place is 5 or more, *round up* by adding 1 to the digit in the specified place (461 rounded to the nearest hundred is 500).
- If the digit in the first place after the specified place is less than 5, *round down* by *not* changing the digit in the specified place (441 rounded to the nearest hundred is 400).

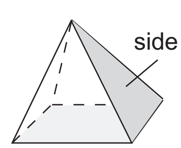
scalene triangle .... a triangle having no congruent sides



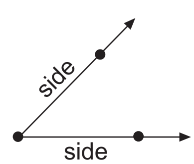
side .... the edge of a polygon, the face of a polyhedron, or one of the rays that make up an angle
Example: A triangle has three sides.



edge of a polygon



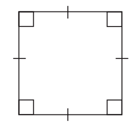
face of a polyhedron



ray of an angle

simplify an expression .... to perform as many of the indicated operations as possible

square .... a rectangle with four sides the same length

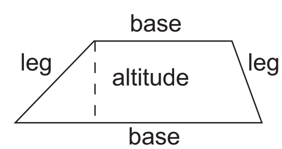


substitute .... to replace a variable with a numeral
Example: $8(a) + 3$
$8(5) + 3$

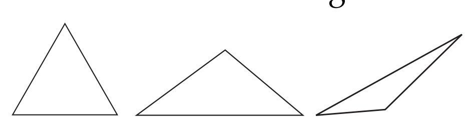
sum .... the result of adding numbers together
Example: In $6 + 8 = 14$,
14 is the sum.

transitive property .... when the first element has a particular relationship to a second element that in turn has the same relationship to a third element, the first has this same relationship to the third element
For any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.
Example:
$5 + 7 = 8 + 4$ and $8 + 4 = 12$,
then $5 + 7 = 12$.
Identity and equality are transitive properties.

trapezoid .... a quadrilateral with just one pair of opposite sides parallel



triangle .... a polygon with three sides; the sum of the measures of the angles is 180°



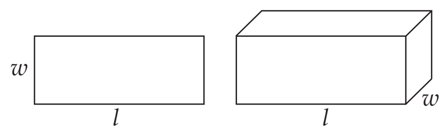
two-dimensional (2-dimensional) .... existing in two dimensions; having length and width

value (of a variable) .... any of the numbers represented by the variable

variable .... any symbol, usually a letter, which could represent a number

whole number .... the numbers in the set {0, 1, 2, 3, 4, …}

width (*w*) .... a one-dimensional measure of something side to side



Unit 3: Examining Formulas and Relationships with Geometric Shapes

Introduction

How can my knowledge of perimeter of polygons help me with algebra?

How can my knowledge of algebra help me apply my knowledge of perimeter?

Lesson One Purpose

- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand and use the real number system. (MA.A.2.4.2)
- Select and justify alternative strategies, such as using properties of numbers, including inverse, identity, distributive, associative, and transitive, that allow operational shortcuts for computational procedures in real-world or mathematical problems. (MA.A.3.4.2)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)
- Use concrete and graphic models to derive formulas for finding perimeter, area, surface area, circumference, and volume of two- and three-dimensional shapes, including rectangular solids, cylinders, cones, and pyramids. (MA.B.1.4.1)
- Relate the concepts of measurement to similarity and proportionality in real-world situations. (MA.B.1.4.3)

- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)
- Describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, tables, and graphs. (MA.D.1.4.1)
- Represent real-world problem situations using finite graphs. (MA.D.2.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)

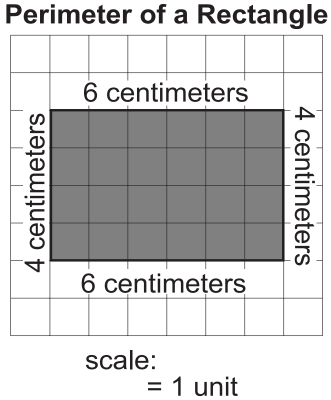
Perimeter—The Distance around a Polygon

If you have walked around a block, walked around a house, or walked around a track, then you have basically walked around the **perimeter (*P*)** of the block, house, or track. *Perimeter* may be determined in a few different ways.

Remember: Perimeter (*P*) means the distance around a **polygon**.

For example, the perimeter of the **rectangle** shown below can be determined in a number of ways. In the next several pages, we will examine three different methods for finding the perimeter of rectangles.

Perimeter of a Rectangle



Practice

Study the following.

Method One

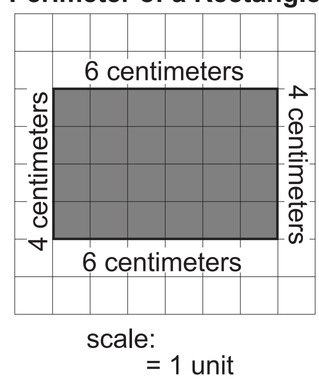
$6 + 4 + 6 + 4 = 20$

The perimeter is 20 centimeters.

This uses the general **formula**:

$l + w + l + w = P$

Perimeter of a Rectangle



where l represents the **length**, w represents the **width**, and P represents the *perimeter*.

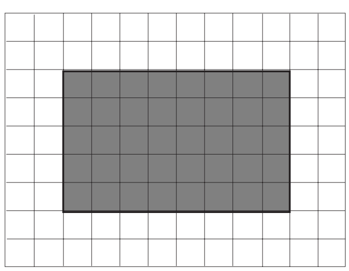
This formula uses the **sum** of the 4 sides to find the perimeter.

We call l, w, and P **variables** because their **values** may vary from one rectangle to another. P will represent the perimeter whether it is 2.5 centimeters, 60 inches, 5 inches, or 40 feet.

Use **Method One** *and the* **formula** *below to determine the* **perimeter** *of each of the following* **rectangles**.

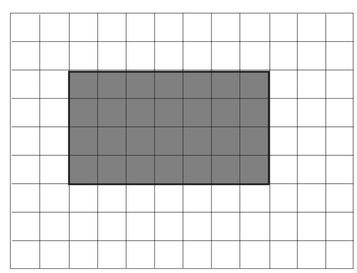
$\boldsymbol{l + w + l + w = P}$

1.



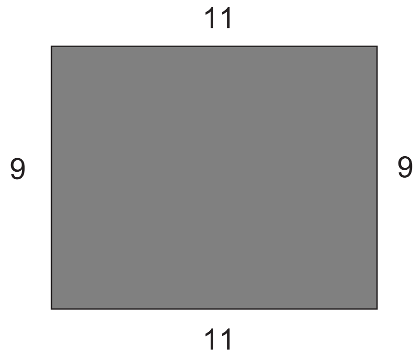
Answer: ____________

2.



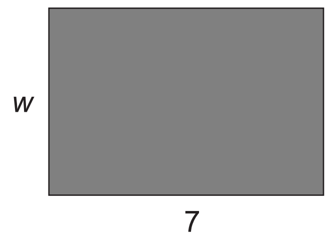
Answer: ____________

3.



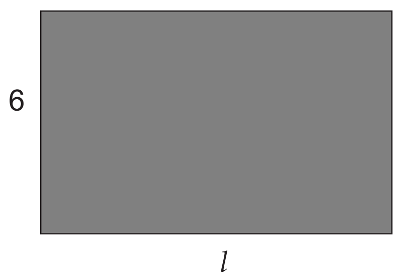
Answer: ______________

5.



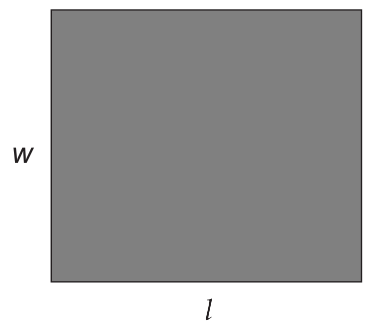
Answer: ______________

4.



Answer: ______________

6.



Answer: ______________

Practice

Study the following.

Method Two

2(6) + 2(4) = 12 + 8 = 20

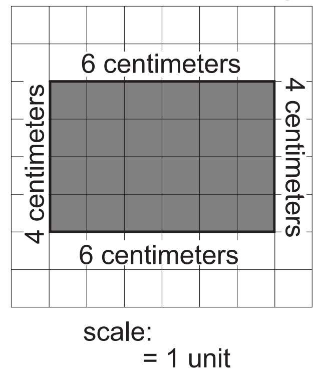
The perimeter is 20 centimeters.

This uses the general formula:

$$2l + 2w = P$$

where l represents the *length*, w represents the *width* and P represents the *perimeter*.

Perimeter of a Rectangle



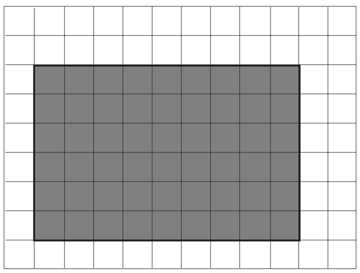
This formula uses the *sum* of twice the length and twice the width to determine the perimeter.

When writing algebraic **expressions** and **equations**, we do not use the "x" symbol for multiplication. When there is no visible operation symbol between a variable and its **coefficient**, it is understood that we multiply.

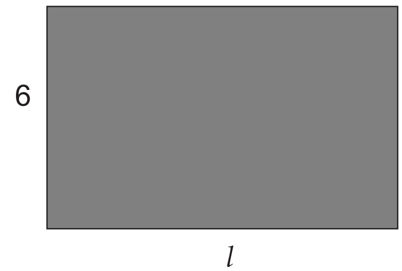
The *coefficient* of the variables l and w is 2, indicating that we multiply the length by 2 and multiply the width by 2. The coefficient of a term is the number part in front of an algebraic expression, signifying multiplication. The coefficient is the numerical factor.

Use **Method Two** *and the* **formula** *below to determine the* **perimeter** *of each of the following* **rectangles***.*

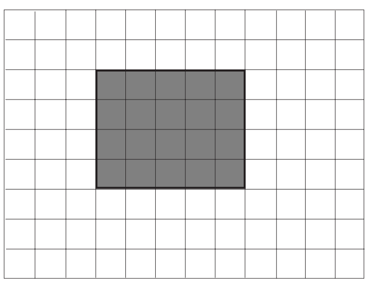
$$2l + 2w = P$$

1. 

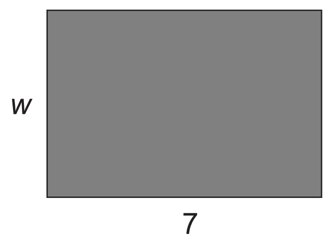
Answer: ____________

4. 

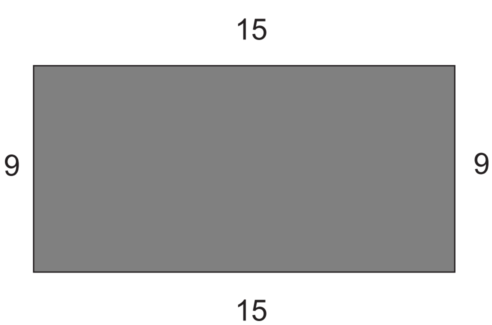
Answer: ____________

2. 

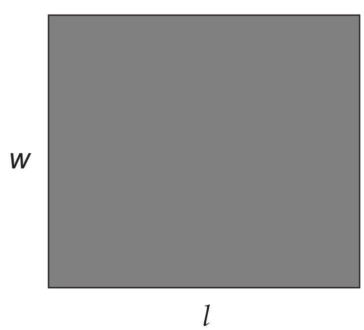
Answer: ____________

5. 

Answer: ____________

3. 

Answer: ____________

6. 

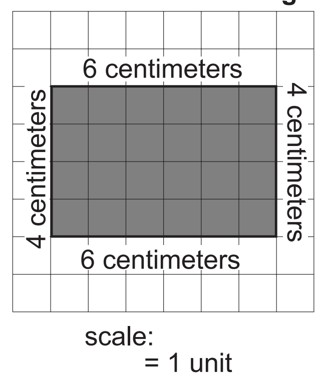
Answer: ____________

Practice

Study the following.

Method Three

Perimeter of a Rectangle



$2(6 + 4) = 2(10) = 20$

The perimeter is 20 cm.

This uses the general formula:

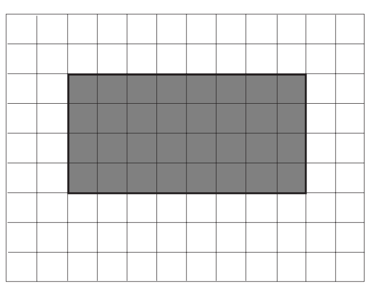
$2(l + w) = P$

where l represents the *length*, w represents the *width*, and p represents the *perimeter*.

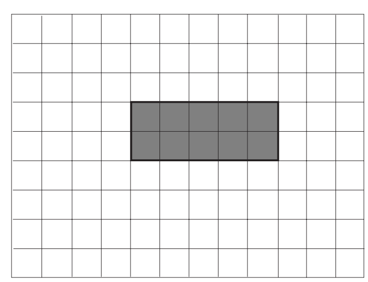
This formula uses twice the *sum* of the length and the width to determine the perimeter.

Use **Method Three** *and the* **formula** *below to determine the* **perimeter** *of each of the following* **rectangles**.

$\mathbf{2(l + w) = P}$

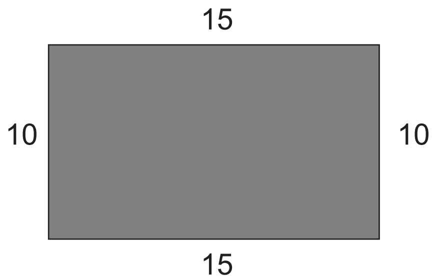
1. 

Answer: ____________

2. 

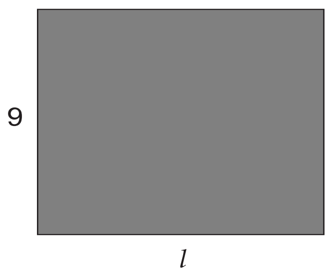
Answer: ____________

3.



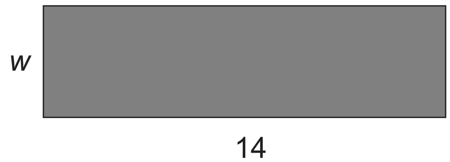
Answer: ______________

4.



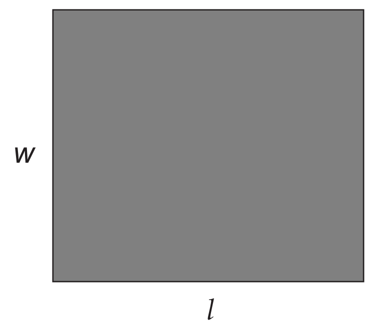
Answer: ______________

5.



Answer: ______________

6.



Answer: ______________

Practice

Match each definition with the correct term. Write the letter on the line provided.

______	1. a parallelogram with four right angles	A. coefficient
______	2. a way of expressing a relationship using variables or symbols that represent numbers	B. equation
______	3. the distance around a polygon	C. expression
______	4. the number part in front of an algebraic term signifying multiplication	D. formula
______	5. any symbol, usually a letter, which could represent a number	E. length (l)
______	6. the result of adding numbers together	F. perimeter (P)
______	7. a one-dimensional measure that is the measurable property of line segments	G. rectangle
______	8. a collection of numbers, symbols, and/or operation signs that stands for a number	H. sum
______	9. any of the numbers represented by the variable	I. value (of a variable)
______	10. a one-dimensional measure of something side to side	J. variable
______	11. a mathematical sentence in which two expressions are connected by an equality symbol	K. width (w)

Comparing Methods One, Two, and Three for Finding Perimeter

Let's now consider why all three formulas used with the three methods produce the same perimeter for the rectangles provided.

First, review the chart below to examine some basic *properties* that will help us work with variables.

Three Methods to Find Perimeter of a Rectangle

Method One $l + w + l + w = P$	**Method Two** $2l + 2w = P$	**Method Three** $2(l + w) = P$
When variables have no visible coefficient, we understand there is an invisible coefficient of 1. We can rewrite this formula as follows: $1l + 1w + 1l + 1w = P$ When terms contain the same variable and the same **exponents**, they are called **like terms**. Sometimes it is helpful to underline *like terms* when we are **simplifying an expression**. $\underline{\mathbf{1l}} + 1w + \underline{\mathbf{1l}} + 1w = P$ The **commutative property** states that the order in which any two numbers are added (or multiplied) does *not* change their sum (or product). This allows us to change the order of the second and third terms: $\mathbf{1l} + 1w + \mathbf{1l} + 1w = P$ $\mathbf{1l} + \mathbf{1l} + 1w + 1w = P$ $\mathbf{2l} + 2w = P$ Compare the formula used in Method One to the formula used in Method Two. The formulas are *equivalent*; you will find that they are the same.	The **distributive property** tells us that you can *distribute* the numbers to write an *equivalent* or equal expression. For all numbers a, b, and c: $a(b + c) = ab + bc$ or $ab + bc = a(b + c)$ $3(7 + 4) = 3(7) + 3(4)$ or $3(7) + 3(4) = 3(7 + 4)$ Think of the distributive property of multiplication as "spreading" things out or *distributing* things out to make it easier to work with—yet making *no* difference in the outcome. $2l + 2w = 2(l + w)$ Now examine the formulas used in Method Two and Method Three. They are also equivalent.	The **transitive property** tells us that when the *first* element (a) has a particular relationship to a *second* element (b) which in turn has the same relationship to a *third* element (c), the *first* element (a) has this same relationship to the *third* element (c). Therefore, for any numbers a, b, and c, the following is true. If $a = b$, and $b = c$, then $a = c$. If $a = b$—Method 1 perimeter formula $\boxed{l + w + l + w = 2l + 2w}$ and $b = c$—Method 2 perimeter formula $\boxed{2l + 2w = 2(l + w)}$ then $a = c$—Method 3 perimeter formula $\boxed{l + w + l + w = 2(l + w)}$ Therefore, in comparing the formulas used in Methods One, Two, and Three, you will find that they are all equivalent.

These formulas can help us establish an important relationship between the perimeter of a rectangle and its dimensions.

$$\text{If } 2l + 2w = P, \text{ then } 1l + 1w = \tfrac{1}{2}P$$

$$\text{If } 2(l + w) = P, \text{ then } 1(l + w) = \tfrac{1}{2}P$$

Experiment to determine if the following is true. If the distributive property of multiplication works over addition, see if it also works over subtraction.

$$a(b - c) = ab - ac.$$

Properties

Addition		Multiplication	
Commutative:	$a + b = b + a$	Commutative:	$ab = ba$
Associative:	$(a + b) + c = a + (b + c)$	Associative:	$(ab)c = a(bc)$
Identity:	0 is the identity. $a + 0 = a$ and $0 + a = a$	Identity:	1 is the identity. $a \bullet 1 = a$ and $1 \bullet a = a$
Addition		**Subtraction**	
Distributive:	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	Distributive:	$a(b - c) = ab - ac$ and $(b - c)a = ba - ca$

Properties of Equality

Reflexive:	$a = a$
Symmetric:	If $a = b$, then $b = a$.
Transitive:	If $a = b$ and $b = c$, then $a = c$.
Substitution:	If $a = b$, then a may be replaced by b.

Other Strategies to Find Dimensions of a Rectangle

Think about This!

A rectangle has a perimeter of 46 centimeters and a width of 8 centimeters. To find its length, there are a number of strategies you could use. See a few of the strategies below.

Four Strategies to Find the Perimeter of a Rectangle

Strategy	Thinking
$2l + 2w = P$ $2l + 2(8) = 46$ $2l + 16 = 46$ $2l + 16 - 16 = 46 - 16$ $2l = 30$ $(2l) \div 2 = 30 \div 2$ $l = 15$	Using the formula, I will **substitute** 8 for the width and 46 for the perimeter and solve the resulting equation for l (length). I do this using step-by-step algebraic equation-solving: • subtract 16 from both sides of the equation • divide both sides of the equation by 2.
$2l + 2w = P$ $2l + 2(8) = 46$ $? + 16 = 46$ $30 + 16 = 46$ $2(15) + 16 = 46$ Since ? represents $2l$, the $1l$ is $\frac{1}{2}$ of 30 or 15.	Using the formula, I will substitute 8 for the width and 46 for the perimeter and solve the resulting equation for l (length). I do this using *intuitive* equation-solving. I *seek* the number that when added to 16 gives me 46. I know the number is 30. I also know that number represents 2 lengths, so one length would be $\frac{1}{2}$ of that or 15.
$1l + 1w = \frac{1}{2}P$ $1l + 8 = \frac{1}{2}(46)$ $1l + 8 = 23$ $1l + 8 - 8 = 23 - 8$ $1l = 15$	I use the fact that the sum of the length and width will be $\frac{1}{2}$ of the perimeter. I represent that in an equation, make *substitutions*, and solve. I subtract 8 from both sides of the equation.
$1l + 1w = \frac{1}{2}P$ $1l + 8 = \frac{1}{2}(46)$ $l + 8 = 23$ $l = 15$	I use the fact that the sum of the length and width is $\frac{1}{2}$ of the perimeter. I *mentally* calculate $\frac{1}{2}$ of 46 is 23 and I think, what number plus 8 is 23?

Do you have a different strategy to share?

Practice

Answer the following. Show all your work.

1. The length of a rectangle is 8 inches and its perimeter is 22 inches. What is its width? Explain how you got your answer.

   Explanation: ____

   ____

   ____

   ____

2. The width of a rectangle is 20 centimeters and its perimeter is 100 centimeters. What is its length? Explain how you got your answer.

   Explanation: ____

   ____

   ____

   ____

3. The perimeter of a rectangle is 30 centimeters. In the table below, list all possible **whole number** lengths and widths for this rectangle.

Remember: If you list a 10 by 5 rectangle, there is no need to list a 5 by 10 rectangle, since they are **congruent (≅)**, or the same shape and size.

$2(10) + 2(5) = 30$
$2(5) + 2(10) = 30.$

Lengths and Widths for a Rectangle with a Perimeter of 30 Centimeters (cm)

Length in cm	Width in cm	Perimeter in cm
		30
		30
		30
		30
		30
		30
		30

4. List the dimensions of the rectangle with the least area.

Answer: ____________________

List the dimensions of the rectangle with the greatest area.

Answer: ____________________

5. Crown molding is being purchased for placement where the 4 walls of a dining room meet the ceiling. The dimensions of the dining room are 14 feet by 17 feet. The crown molding cost $7.80 per running foot. What is the cost of the crown molding for the dining room?

Answer: ____________________

Guess and Check to Find Dimensions of a Rectangle

Think about This!

The length of a rectangle is 12 more inches than its width. The perimeter of the rectangle is 44 inches. What are the dimensions of the rectangle?

This problem could be solved by *guess and check*.

- If width is 1, length would be 13 and perimeter would be 28 inches.
- If width is 2, length would be 14 and perimeter would be 32 inches.
- If width is 3, length would be 15 and perimeter would be 36 inches.
- If width is 4, length would be 16 and perimeter would be 40 inches.
- If width is 5, length would be 17 and perimeter would be 44 inches.

Applying Formulas to Find Dimensions of a Rectangle

The problem above could also be solved by representing the width as w, the length as $w + 12$, and the perimeter as 44. We could then write a formula to solve. See below.

Formulas to Find Dimensions of a Rectangle with a Length of 12 More Inches Than Its Width and a Perimeter of 44 Inches

$l + w + l + w = P$ $(w + 12) + w + (w + 12) + w = 44$ $4w + 24 = 44$ $4w + 24 - 24 = 44 - 24$ $4w = 20$ $\frac{4w}{4} = \frac{20}{4}$ $w = 5$ If the width is 5, the length is 12 + 5 or 17.	$2l + 2w = P$ $2(w + 12) + 2(w) = 44$ $2w + 24 + 2w = 44$ $4w + 24 = 44$ $4w + 24 - 24 = 44 - 24$ $4w = 20$ $\frac{4w}{4} = \frac{20}{4}$ $w = 5$ If the width is 5, the length is 12 + 5 or 17.
$2(l + w) = P$ $2(w + 12 + w) = 44$ $2w + 24 + 2w = 44$ $4w + 24 = 44$ $4w + 24 - 24 = 44 - 24$ $4w = 20$ $\frac{4w}{4} = \frac{20}{4}$ $w = 5$ If the width is 5, the length is 12 + 5 or 17.	$l + w = \frac{1}{2}P$ $w + 12 + w = \frac{1}{2}(44)$ $2w + 12 = 22$ $2w + 12 - 12 = 22 - 12$ $2w = 10$ $\frac{2w}{2} = \frac{10}{2}$ $w = 5$ If the width is 5, the length is 12 + 5 or 17.

Do you have a different strategy to share?

Practice

Answer the following. Show all your work.

1. The width of a rectangle is 4 less than its length and the perimeter is 20 centimeters. What are the dimensions of the rectangle? Explain the strategy you chose.

   Explanation: ____

   ____

   ____

   ____

Practice

Use the list below to write the correct term for each definition on the line provided.

commutative property	**exponent**	**substitute**
congruent (≅)	**like terms**	**transitive property**
distributive property	**simplify an expression**	**whole number**

_________________________ 1. the numbers in the set {0, 1, 2, 3, 4, …}

_________________________ 2. the order in which two numbers are added or multiplied does *not* change their sum or product
Example: 2 + 3 = 3 + 2 *or* 4 x 7 = 7 x 4

_________________________ 3. the number of times the base occurs as a factor

_________________________ 4. figures or objects that are the same shape and size

_________________________ 5. terms that have the same variables and the same corresponding exponents

_________________________ 6. the product of a number and the sum or difference of two numbers is equal to the sum or difference of the two products
Example: $x(a + b) = ax + bx$

_________________________ 7. when the first element has a particular relationship to a second element that in turn has the same relationship to a third element, the first has this same relationship to the third element
Example: For any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.

_________________________ 8. to replace a variable with a numeral
Example: $8(a) + 3$
$8(5) + 3$

_________________________ 9. to perform as many of the indicated expressions as possible