Lesson Two Purpose

- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand and use the real number system. (MA.A.2.4.2)
- Select and justify alternative strategies, such as using properties of numbers, including inverse, identity, distributive, associative, and transitive, that allow operational shortcuts for computational procedures in real-world or mathematical problems. (MA.A.3.4.2)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)
- Use concrete and graphic models to derive formulas for finding perimeter, area, surface area, circumference, and volume of two- and three-dimensional shapes, including rectangular solids, cylinders, cones, and pyramids. (MA.B.1.4.1)
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)
- Describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, tables, and graphs. (MA.D.1.4.1)
- Represent real-world problem situations using finite graphs. (MA.D.2.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)

Simplifying Expressions

The following strategy is frequently used to simplify expressions.

Since we know $a - b = a + (-b)$,

- change each subtraction problem to an addition problem by adding the opposite of the number which follows the subtraction sign
- combine *like terms* (terms that have the same variable) by adding.

Simplify

$2a + 3 - 6a$

like terms

$2a + 3 - 6a = 2a + 3 + (-6a)$

$= -4a + 3$

Simplify

$8b + 7 - b - 6$

like terms

$8b + 7 - b - 6 = 8b + 7 + (-1b) + (-6)$

$= 7b + 1$

like terms

The following practice allows you to extend your knowledge of perimeter as you use some skills of algebra.

Practice

The following expressions have **like terms**. *Combine the like terms to* **simplify the expression**.

Remember: x is the same as $1x$.

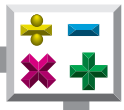
1. $x + y + x + y =$
2. $2a + b + a + 3b =$
3. $m + 2n + 3n + p =$
4. $r + 4s + 2r + 3r =$
5. $s + 4s - 2s + 3r - 2r =$

Use the **distributive property** *to write an* **equivalent expression** *for each of the following.*

Remember: The distributive property allows us to write an equivalent expression for

$a(b + c)$ **as** $ab + ac$ and $ab + ac$ **as** $a(b + c)$ or $3(5 + 3)$ **as** $3(5) + 3(3)$ and $3(5) + 3(3)$ **as** $3(5 + 3)$

6. $4(2a + 3b) =$
7. $2(6x + y) =$
8. $3(5m + 2n) =$
9. $9x + 18z =$
10. $24(3r + 7b) =$
11. $4c + 10f =$
12. $8b + 12s =$
13. $20v + 70d =$



Think about This!

When we mentally compute, we often use the distributive property. Consider this example:

$$37(12) = 37(10 + 2) = 37(10) + 37(2) = 370 + 74 = 444$$

$$23(19) = 23(20 - 1) = 23(20) - 23(1) = 460 - 23 = 437$$

Use the **distributive property** *to mentally compute the following products.*

14. $22(11) =$

15. $15(19) =$

16. $45(9) =$

17. $32(22) =$

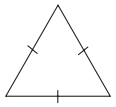
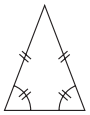
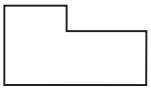
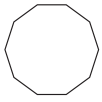
18. $240(12) =$

Polygons

So far, we have worked with rectangles and **squares** in this unit. These shapes are considered *polygons* (having at least three **sides** that are **line segments (—)** connected at their **endpoints**). There are many other shapes included in the broad category of polygons.

This chart identifies many geometric shapes that are polygons.

Polygons

Name of Polygon	Description	Examples
rectangle	4 sides 4 right angles	
triangle	3 sides	 **equilateral triangle**  **isosceles triangle**  **scalene triangle**
square	4 sides the same length 4 right angles (A square is also a rectangle.)	
pentagon	5 sides	 
hexagon	6 sides	 
heptagon	7 sides	
octagon	8 sides	
nonagon	9 sides	
decagon	10 sides	

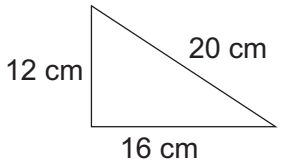
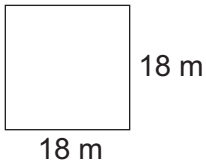
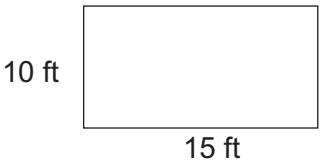
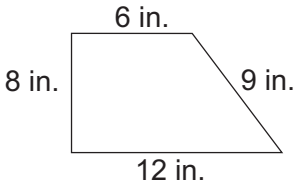
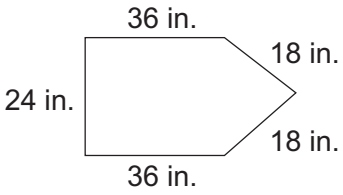
Perimeter of Polygons

A **regular polygon** has *sides* of equal lengths and angles of equal measurements, thus all sides and angles are *congruent*. This knowledge will aid us in finding the perimeter of *regular polygons*.

To compute the perimeter (P) of a polygon, we determine the distance around the polygon. Lengths of sides and various formulas are used. *Sides* are the edges of **two-dimensional** geometric figures. *Two-dimensional* figures have two dimensions: length (l) and width (w).

We have used formulas to find perimeter of rectangles in this unit. For other polygons in this unit, such as triangles, trapezoids, and pentagons, we simply add the lengths of all the sides (s) to get perimeter (P). See some examples on the following page.

Formulas for Finding the Perimeter of Polygons

Examples	Formulas
triangle  	Perimeter = the sum of the lengths of the three sides—*a*, *b*, and *c* $P = a + b + c$ $P = 12 \text{ cm} + 16 \text{ cm} + 20 \text{ cm}$ $P = 48$ centimeters The perimeter is 48 centimeters.
square  	Perimeter = 4 times the length (l) of one side (s = side length) $P = 4s$ $P = 4(18 \text{ m})$ $P = 72 \text{ m}$ The perimeter is 72 meters.
rectangle  	Perimeter = 2 times the length (l) plus 2 times the width (w) $P = 2l + 2w$ $P = 2(15 \text{ ft}) + 2(10 \text{ ft})$ $P = 30 \text{ ft} + 20 \text{ ft}$ $P = 50 \text{ ft}$ The perimeter is 50 feet.
trapeziod  	Perimeter = the sum of the lengths of all the sides $P = s_1 + s_2 + s_3 + s_4$ $P = 8 \text{ in.} + 6 \text{ in.} + 9 \text{ in.} + 12 \text{ in.}$ $P = 35 \text{ in.}$ The perimeter of the trapezoid is 35 inches.
pentagon  	Perimeter = the sum of the lengths of all the sides $P = s_1 + s_2 + s_3 + s_4 + s_5$ $P = 24 \text{ in.} + 36 \text{ in.} + 18 \text{ in.} + 18 \text{ in.} + 36 \text{ in.}$ $P = 132 \text{ in.}$ The perimeter of the pentagon is 132 inches.

Practice

Complete the following table.

Perimeter of Regular Polygons

Number of Congruent Sides and Angles	Name of Polygon	Length of a Side	Perimeter	General Formula for Perimeter
3	equilateral triangle	4 inches	12 inches	$s + s + s = P$ or $3s = P$
4		2.5 inches		$s + s + s + s = P$ or $4s = P$
5	pentagon		30 centimeters	
6		2 feet		
7	heptagon	5 centimeters		
8			72 millimeters	$s + s + s + s + s + s + s + s = P$ or $8s = P$
9	nonagon	10 inches		
10			15 inches	

Perimeter of Isosceles Triangles

All triangles are not regular polygons. A triangle with *two* congruent sides and *two* congruent angles is called an *isosceles triangle*. If the side lengths in an isosceles triangle are 4 inches, 4 inches, and 3 inches, the perimeter would be 11 inches.

$$4 + 4 + 3 = 11$$

The general formula could be

$$\boxed{a + a + b = P}$$

where a represents the length of each of the 2 congruent sides, b represents the length of the third side and P represents the perimeter.

Combining like terms, we could also write this formula as

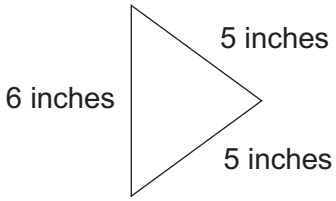
$$\boxed{2a + b = P}$$

Practice

Use the **formula** *below to find the* **perimeter** *of each of the following* **isosceles triangles**. *Combine like terms to* **simplify the expression**. *Explain how you got your answer. Show your work.*

$$a + a + b = P$$
or
$$2a + b = P$$

1.



Answer: _____

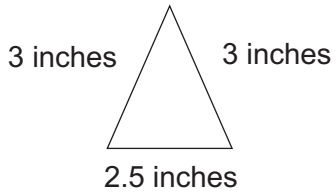
Explanation: _____

_____

_____

_____

2.



Answer: _____

Explanation: _____

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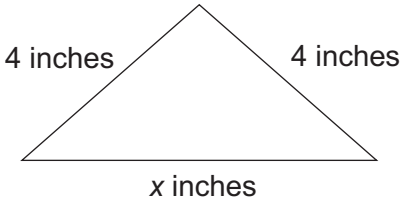
3.



Answer: _____

Explanation: _____

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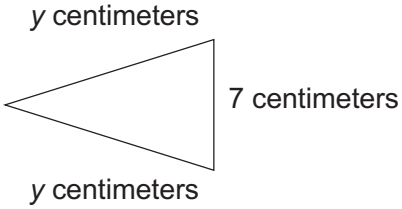
4.



Answer: _____

Explanation: _____

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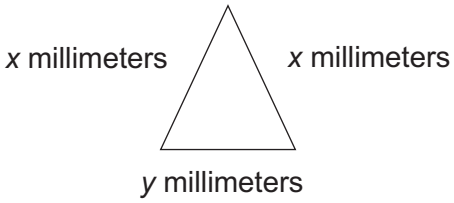
5.



Answer: _____

Explanation: _____

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_____

6. If an isosceles triangle has two sides with a length of 4 inches and a perimeter of 13 inches, what is the length of the *third* side?

   Answer: _____ inches

7. If the longest side of an isosceles triangle has a length of 12 centimeters and the perimeter of the triangle is 30 centimeters, what is the length of *one* of the two congruent sides?

   Answer: _____ centimeters

Perimeter of Scalene Triangles

A triangle with *no* congruent sides and *no* congruent angles is called a *scalene triangle*.

If the side lengths of a *scalene triangle* are 4 inches, 5 inches, and 8 inches, the perimeter will be 17 inches.

$$4 + 5 + 8 = 17$$

The general formula could be

$$\boxed{a + b + c = P}$$

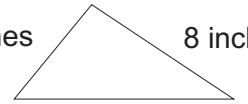
where a, b, and c represent the three different side lengths and P represents the perimeter.

Practice

Use the **formula** *below to find the* **perimeter** *of each of the following* **scalene triangles**. *Combine like terms to* **simplify the expression**. *Explain how you got your answer. Show your work.*

$$a + b + c = P$$

1. 5 inches, 8 inches, 10 inches



Answer: _____

Explanation: _____

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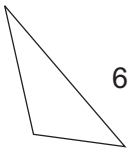
2. 5 inches, 6 inches, 2 inches



Answer: _____

Explanation: _____

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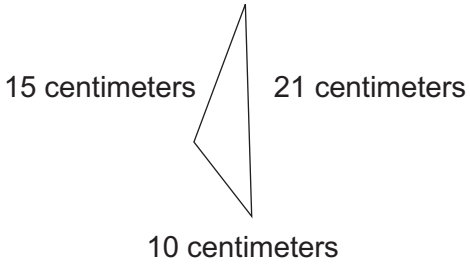
3.



Answer: _____

Explanation: _____

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_____

4. The perimeter of a regular polygon is 40 centimeters and the length of each side is a whole number. Name all polygons it might be if it has less than 12 sides. Explain.

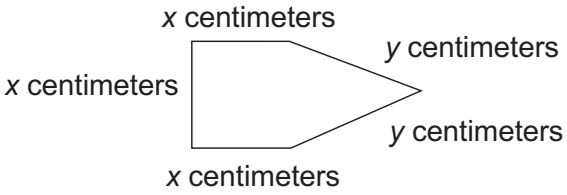
Answer: _____

_____

Explanation: _____

_____

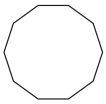
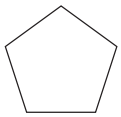
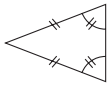
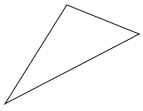
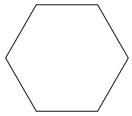
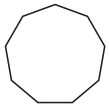
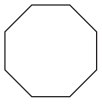
5. The following pentagon is *not* a regular polygon since all sides and all angles are *not* congruent. Find the perimeter of this pentagon.



Answer: _____

Practice

Match each **polygon shape** *with the correct* **polygon name**. *Write the letter on the line provided.*

polygon shape		**polygon name**
_____ 1.		A. decagon
_____ 2.		B. equilateral triangle
_____ 3.		C. heptagon
_____ 4.		D. hexagon
_____ 5.		E. isosceles triangle
_____ 6.		F. nonagon
_____ 7.		G. octagon
_____ 8.		H. pentagon
_____ 9.		I. rectangle
_____ 10.		J. scalene triangle
_____ 11.		K. square

Practice

Use the list below to write the correct term for each definition on the line provided.

equilateral triangle	**scalene triangle**
isosceles triangle	**two-dimensional**
regular polygon	

_____ 1. a triangle having no congruent sides

_____ 2. a polygon that is both *equilateral* (all sides congruent) and *equiangular* (all angles congruent)

_____ 3. a triangle with three congruent sides

_____ 4. existing in two dimensions; having length and width

_____ 5. a triangle with two congruent sides and two congruent angles