



Lesson Three Purpose

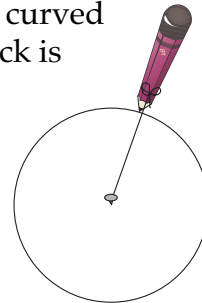
- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand and use the real number system. (MA.A.2.4.2)
- Select and justify alternative strategies, such as using properties of numbers, including inverse, identity, distributive, associative, and transitive, that allow operational shortcuts for computational procedures in real-world or mathematical problems. (MA.A.3.4.2)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)
- Use concrete and graphic models to derive formulas for finding perimeter, area, surface area, circumference, and volume of two- and three-dimensional shapes, including rectangular solids, cylinders, cones, and pyramids. (MA.B.1.4.1)
- Relate the concepts of measurement to similarity and proportionality in real-world situations. (MA.B.1.4.3)
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)
- Represent and apply geometric properties and relationships to solve real-world and mathematical problems including ratio and proportion. (MA.C.3.4.1)



- Describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, tables, and graphs. (MA.D.1.4.1)
- Represent real-world problem situations using finite graphs. (MA.D.2.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)

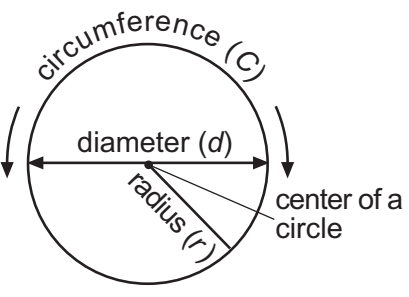
Circles

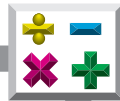
Suppose that you tie a string to a thumbtack and push it into a piece of cardboard. Next you tie a marker or pencil to the other end of the string and you move the string around the thumbtack. The curved line drawn by the marker or pencil is a **circle**. The thumbtack is the **center of the circle**. The *center of the circle* is the **point** from which all the *points* on the circle are the same distance.



A *circle* is the set of all points that are the same distance from the center. The distance of any line segment drawn from the center to any point on the circle is called a **radius (r)**. A **diameter (d)** of a circle is the distance of a *line segment* ($-$) from one point of the circle to another, through the center.

The distance around a circle is called the **circumference (C)** while the distance around polygons is called the *perimeter*.

Circles	
	<p>circumference (C) - the distance around a circle or the perimeter (P) of a circle</p> <p>diameter (d) - a line segment that passes through the center of a circle to another point on a circle</p> <p>center of a circle - the point from which all points on a circle are the same distance</p> <p>radius (r) - any line segment from the center of a circle to a point on a circle</p>



Circumference—The Distance around a Circle

You may have had the opportunity in the past to measure the *circumference* of a variety of circular objects along with the *diameter* for each. You probably noticed that the circumference was a bit more than 3 times the diameter for each of your circular objects. You then might have used the following formula for circumference

$$C = \pi d$$

where C represents circumference, π is the Greek letter π (**pi**) and represents approximately 3.14, and d represents diameter.

Another equivalent formula sometimes seen is

$$C = 2\pi r$$

where C represents circumference and r represents *radius*. We know these two formulas are equivalent because the length of the diameter in a given circle is twice the length of a radius in that circle.

pi (π)

If you look on a scientific calculator, you will find a key for π . If you press it, you will probably get 3.141592654.

π is neither 3.14 nor the long number which the calculator gave us. One of the great challenges faced by mathematicians through the ages has been trying to find an *exact value* for π . However, there isn't one! In this section we will use 3.14 for π . Our answers will be *approximate* (\approx).

If the π key on the calculator is used, the answer will be more accurate but still approximate.

Note: The fraction $\frac{22}{7}$ is also used for π .

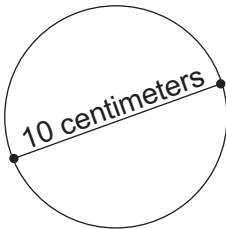


Practice

Use the **formula** below to find the **circumference** of each of the following circles. You may use **3.14** or $\frac{22}{7}$ for π , whichever is more convenient. Show all your work. **Round** to the nearest **whole number**.

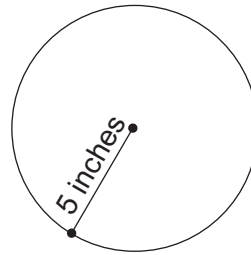
$$C = \pi d \text{ or } C = 2\pi r$$

1.



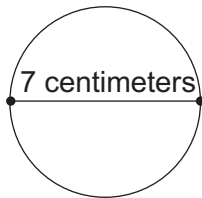
Answer: _____

4.



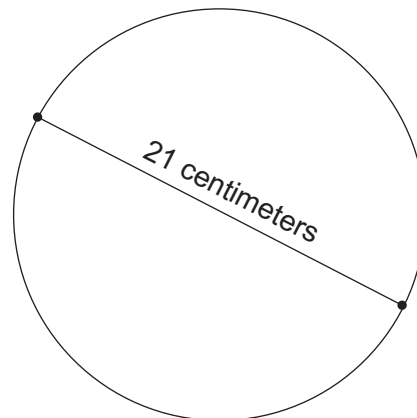
Answer: _____

2.



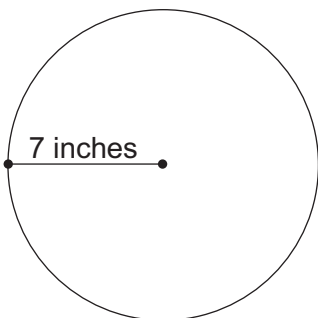
Answer: _____

5.

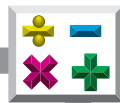


Answer: _____

3.



Answer: _____

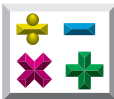


6. Choose three of the circles in numbers 1-5 to explain how you might use mental math to determine the circumference rather than using a calculator or paper and pencil.

Explanation for circle number _____ :

Explanation for circle number _____ :

Explanation for circle number _____ :



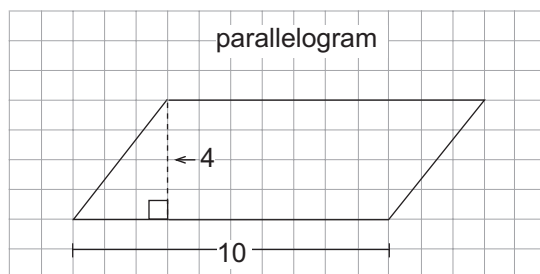
Finding the Perimeter of Non-Rectangular Parallelograms Using the Pythagorean Theorem

You may have had the opportunity in the past to determine the perimeter of *non-rectangular parallelograms*. A *parallelogram* is a **quadrilateral** (four-sided polygon) with two pairs of **parallel** (\parallel) sides.

If the side lengths were provided, you simply found their sum.

If the side lengths were *not* provided you likely estimated, used string and/or a ruler, or you may have used the **Pythagorean theorem**.

Consider the following parallelogram.



By *counting*, we can see that the length of the base is 10 units and that the height is 4 units. However, we do not know the length of the shorter side. To find the length of the shorter side of the parallelogram, we can sketch in a **right triangle** and use the *Pythagorean theorem*, which tells us that in a *right triangle*,

$$a^2 + b^2 = c^2$$

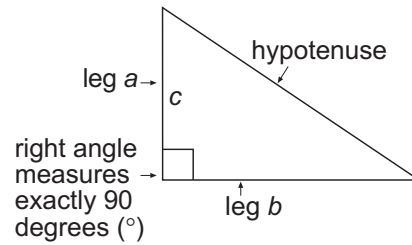
where a and b represent the lengths of the two shorter sides called the **legs** and c represents the length of the longest side called the **hypotenuse**.



The Pythagorean Theorem

Pythagorean theorem: In a right triangle, the sum of the square of the lengths of the legs a and b equals the square of the length of the hypotenuse c .

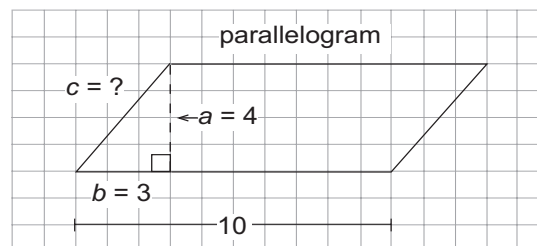
Algebraically: $a^2 + b^2 = c^2$



Remember: To *square* a number, multiply it by itself.

Example: The square of 6 or $6^2 = 6 \times 6 = 36$.

Now, to determine the perimeter of the parallelogram, we need to determine the dimension of the shorter side.



The **height** (h) of the triangle and parallelogram is 4. The **base** (b) of the parallelogram is 10. The base of the triangle is 3.



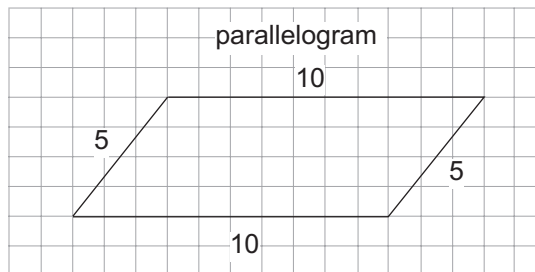
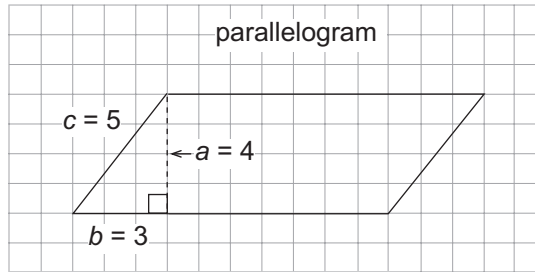
Let's substitute the two shorter sides (legs) into the Pythagorean theorem below.

$$\begin{aligned}a^2 + b^2 &= c^2 \\4^2 + 3^2 &= c^2 \\16 + 9 &= c^2 \\25 &= c^2 \\5 &= c\end{aligned}$$

Now we can determine the perimeter of the parallelogram. Find the sum of all sides.

$$\begin{aligned}5 + 5 + 10 + 10 &= P \\2(5) + 2(10) &= P \\10 + 20 &= P \\30 &= P\end{aligned}$$

So the perimeter of the parallelogram is 30.





Practice

Find the **perimeter** of each of the following **parallelograms**. Show all your work.

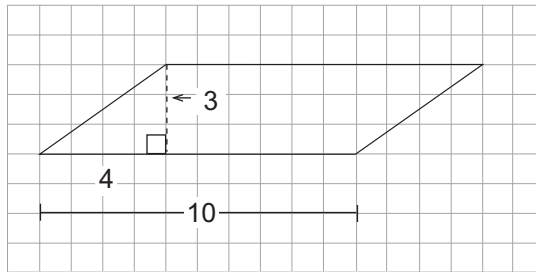
Pythagorean theorem

$$a^2 + b^2 = c^2$$

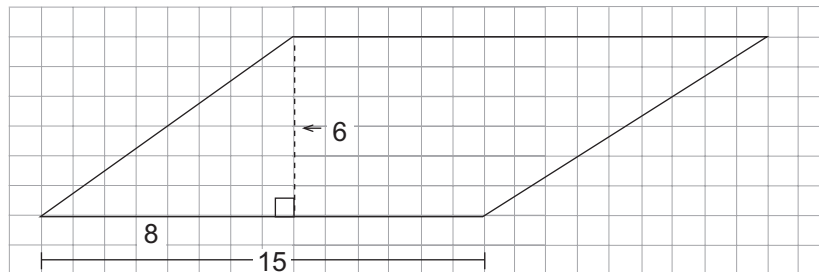
perimeter of a parallelogram

sum of all sides

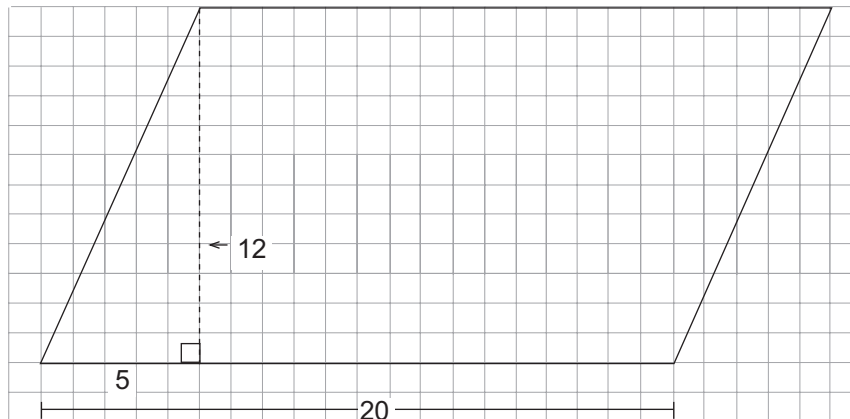
1.



2.



3.

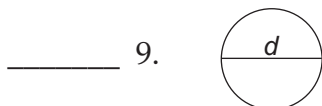
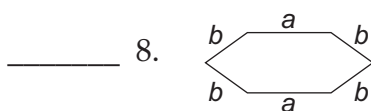
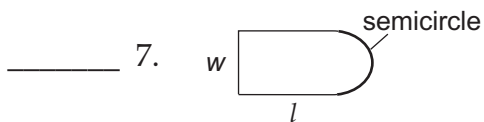
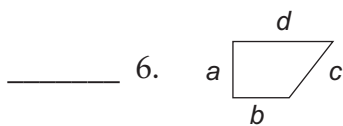
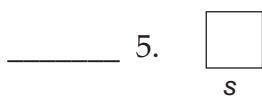
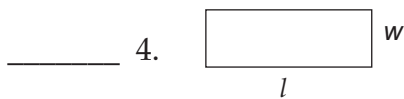
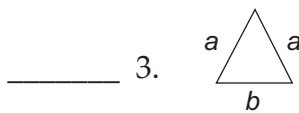
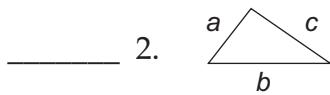
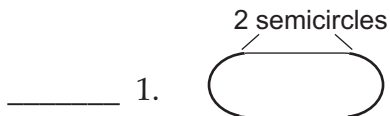




Practice

Match the **shapes** with the correct **formula** for finding **perimeter** or **circumference**. Write the letter on the line provided.

Shapes



Formulas

A. $P = 2l + 2w$

B. $P = 2l + w + \frac{1}{2}\pi d$

C. $P = a + b + c$

D. $C = \pi d$

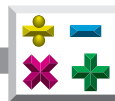
E. $P = 2l + \pi d$

F. $P = a + b + c + d$

G. $P = 2a + b$

H. $P = 4s$

I. $P = 2a + 4b$



Practice

Match each definition with the correct term. Write the letter on the line provided.

- | | | |
|-------|--|----------------------------|
| _____ | 1. the longest side of a right triangle | A. circumference (C) |
| _____ | 2. if $a = b$ and $b = c$, then $a = c$ | B. congruent (\approx) |
| _____ | 3. an irrational number with common approximations of either 3.14 or $\frac{22}{7}$ | C. diameter (d) |
| _____ | 4. $x(a + b) = ax + bx$ | D. distributive property |
| _____ | 5. a line segment from any point on the circle passing through the center to another point on the circle | E. hypotenuse |
| _____ | 6. the distance around a circle | F. leg |
| _____ | 7. in a right triangle, one of the two sides that form the right angle | G. octagon |
| _____ | 8. $c^2 = a^2 + b^2$ | H. parallelogram |
| _____ | 9. a quadrilateral with two pairs of parallel sides | I. pi (π) |
| _____ | 10. to replace a variable with a numeral | J. Pythagorean theorem |
| _____ | 11. a line segment extending from the center of a circle or sphere to a point on the circle or sphere | K. radius (r) |
| _____ | 12. a polygon with eight sides | L. substitute |
| _____ | 13. figures or objects that are the same shape and size | M. transitive property |

