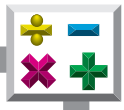
Lesson Two Purpose

- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand and use the real number system. (MA.A.2.4.2)
- Select and justify alternative strategies, such as using properties of numbers, including inverse, identity, distributive, associative, and transitive, that allow operational shortcuts for computational procedures in real-world or mathematical problems. (MA.A.3.4.2)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)
- Use concrete and graphic models to derive formulas for finding rate, distance, time, and angle measurements. (MA.B.1.4.2)
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)
- Describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, tables, and graphs. (MA.D.1.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)



Using What We Know about Triangles to Solve Algebraic Equations

What do we know about all **triangles**? We know

- *triangle*s are **polygons** with three **sides**.
- triangles are closed **plane** figures formed by three **line segments (—)**.
- triangles have straight *sides* that do not cross.
- triangles are classified as either by the **measure (m) of their angles (∠)** or by the **lengths (*l*)** of their sides.
- the sum of the measures (m) of the three **angles (∠)** in any triangle is 180 **degrees (°)**.

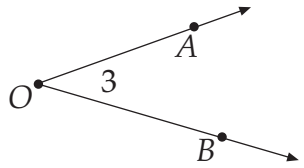
To more fully understand triangles, let's review what we know about *angles*.

Measuring and Classifying Angles

The *sides* of an angle are formed by two **rays (⟶)** extending from a common endpoint called the **vertex**.

Naming an Angle

Consider the following figure.



Remember: The symbol ∠ indicates an angle.

You can name an angle in three ways:

- using a three-letter name in this order: **point** on one *ray*; *vertex*; *point* on other ray, such as ∠*AOB or* ∠*BOA*
- using a one-letter name: vertex, if there is only one angle with this vertex in the diagram, such as ∠*O*
- using a numerical name if the number is within the rays of the angle, such as ∠3.

The angle is formed by rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$. The rays are portions of **lines** (⟷) that begin at a point and go on forever in one direction.

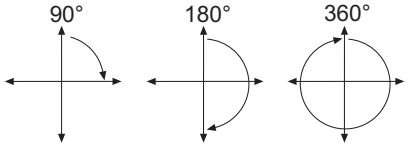
The point *O*, which is the same endpoint for $\overrightarrow{OA}$ and $\overrightarrow{OB}$, is the *vertex* of angle *AOB*. When using three letters to name an angle, the *vertex* letter is listed in the middle.

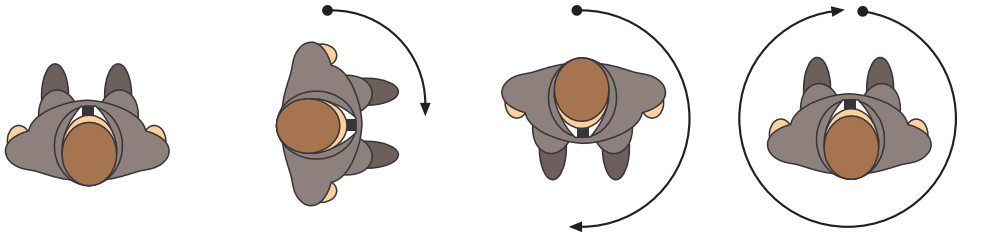
The measure of ∠*O* is written as m∠*O*. Sometimes two (or more) angles have the same measure. When two angles have the same measure, they are **congruent (≅)**.

Measuring an Angle

What do we know about the measure of an angle? The *measure (m) of an angle* (∠) is described in *degrees*. When you turn around and face backward, you could say you "did a 180." If you turn all the way around, it is a 360. An angle is a turn around a point. The size of an angle is the measure of how far one side has turned from the other side.

0°	=	no turn
90°	=	right-angle turn
180°	=	straight backward
360°	=	back facing forward again

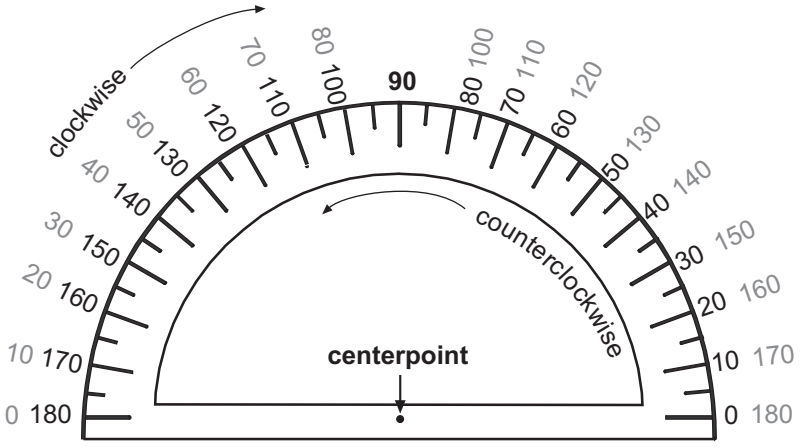




0° = no turn | 90° = right-angle turn | 180° = straight backward | 360° = back facing forward again

Using Protractors to Measure Angles

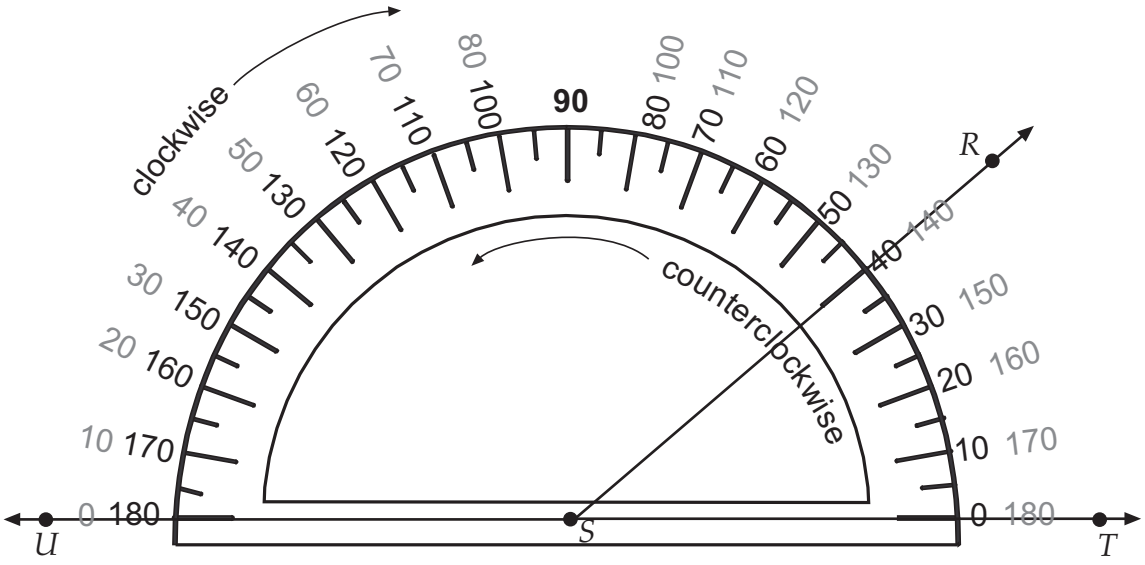
Protractors are marked from 0 to 180 degrees in both a clockwise manner and a counterclockwise manner. We see 10 and 170 in the same position. We see 55 and 125 in the same position. If we estimate the size of the angle before using the *protractor*, there is no doubt which measure is correct.



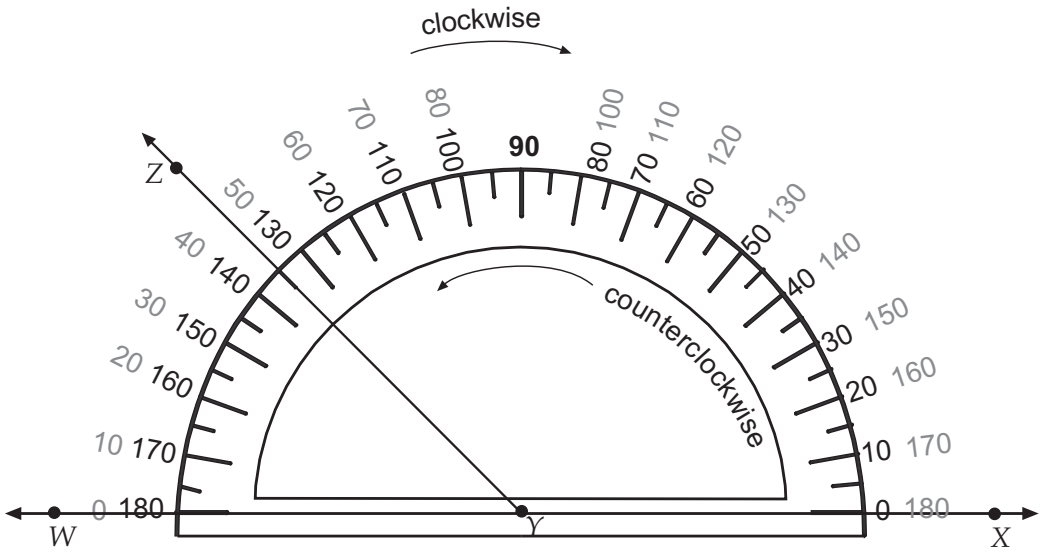
When using a protractor, make sure the vertex is lined up correctly and that one *ray* (→) passes through the zero measure. A straightedge is often helpful to extend a ray for easier reading of the measure.

A *protractor* is used to measure angles. Follow these steps to use a protractor.

1. Place the centerpoint of the protractor on the vertex of the angle.
2. Line up the protractor's 0 degree line with one side of the angle.
3. Read the measure of the angle where the other side crosses the protractor.



- The measurement of $\angle TSR$ is _____40°_____ .
  40° is read 40 degrees.
- The measurement of $\angle USR$ is _____140°_____ .



- The measurement of $\angle XYZ$ is ____135°____ .
- The measurement of $\angle WYZ$ is ____45°____ .

Naming Different Size Angles

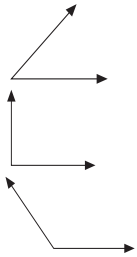
Angles are named for the way they relate to 90 degrees and 180 degrees.

acute angle = < 90° and > 0°

right angle = 90°

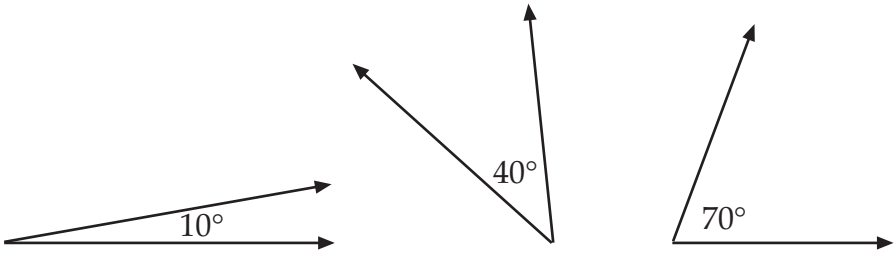
obtuse angle = > 90° and < 180°

straight angle = 180°



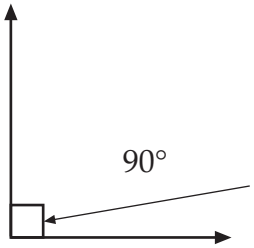
An *acute angle* measures greater than 0° but less than 90°.

Examples:



A *right angle* measures exactly 90°.

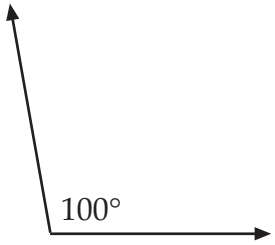
Example:

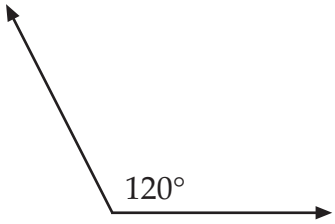


The little box is used to indicate a right angle.

An *obtuse angle* measures greater than 90° but less than 180°.

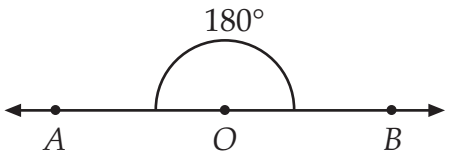
Examples:





A *straight angle* measures exactly 180°.

Example:

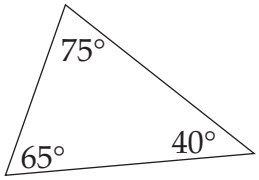


$\angle AOB$ has vertex O and the measure of $\angle AOB$ is 180°. In this case, we need to use three letters to name the angle.

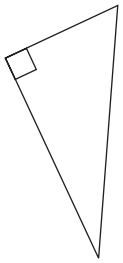
Classifying Triangles by Their Angles or by Their Sides

Triangles Classified by Their Angles—Acute, Right, Obtuse, and Equiangular

An **acute triangle** contains *all* acute angles with measures less than 90°.

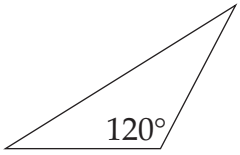


A **right triangle** contains *one* right angle with a measure of 90°.

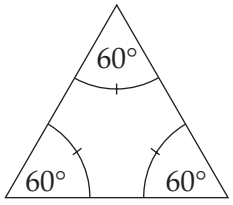


Note: right triangles are marked ⊾

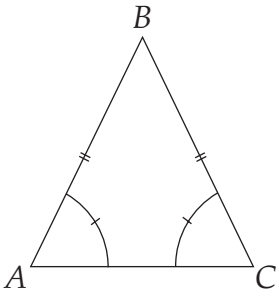
An **obtuse triangle** contains *one* obtuse angle with a measure of more than 90° but less than 180°.



An **equiangular triangle** contains *all* equal angles, each with a measure of 60°.



Note: The same number of *tick* or *slash marks* are used to denote *angles* or *sides* with the same measure. *Arcs* with the same number of tick marks are also used to show *angles* with the same measure.

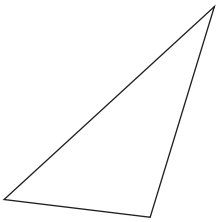


$m\angle A = m\angle C$

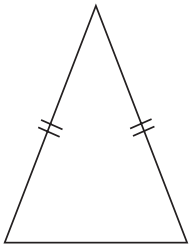
$\overline{AB} = \overline{BC}$

Triangles Classified by the Lengths of Their Sides—Scalene, Isosceles, and Equilateral

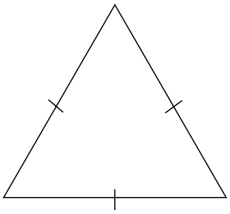
A **scalene triangle** has *no* congruent sides—no sides are the same length.



An **isosceles triangle** has at least *two* congruent sides—two or more sides are the same length.

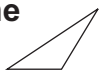


An **equilateral triangle** has *three* congruent sides—*all* sides are the same length.



So, all triangles may be classified by their angles (acute, right, obtuse, or equiangular), by their sides (equilateral, isosceles, scalene), or both. See the chart below.

Triangles

Classification	Acute < 90° and > 0°	Right = 90°	Obtuse > 90° and <180°	Equiangular all = 60°
Equilateral 	✓			✓
Isosceles 	✓	✓	✓	
Scalene 	✓	✓	✓	

< means less than > means greater than

As you see in the above chart, a right triangle *may be* either isosceles or scalene, but is *never* acute, equilateral, or obtuse.

Practice

Solve the following.

1. In scalene triangle ABC, the measure (m) of $\angle B$ is

   - 16 degrees more than the measure of $\angle A$ and
   - 2 degrees more than the measure of $\angle C$.

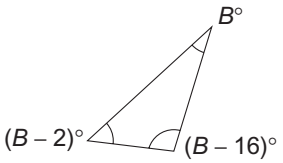
   What is the measure of $\angle B$?

   You may use the setup provided below or you may create your own way to represent the angle measures. Show your work.

   Remember: The sum of the measure of the angles in any triangle is 180°

   measure of $\angle B = \quad B$
   measure of $\angle A = \quad B - 16$
   measure of $\angle C = \quad B - 2$

   $B + B - 16 + B - 2 = 180$

   scalene triangle—has no congruent angles

   measure of $\angle B =$ _____

   Check your work.

   The measure of the angles are as follows:

   $\angle B = \quad B \quad =$ _____
   $\angle A = \quad B - 16 =$ _____
   $\angle C = \quad B - 2 \quad =$ _____
   } 180°

2. In scalene triangle *BCD*, the measures of the angles are **consecutive multiples** of 4 where $\angle B$ is the smallest angle and $\angle D$ is the largest. Find the measure of each angle. Show your work.

   Hint: *Consecutive multiples* of 4 include 4, 8, 12, 16 and so on. Each is 4 more than the previous one, going *in order*.

   Try the following representation first.

   measure of $\angle B =$ B
   measure of $\angle C =$ $B + 4$
   measure of $\angle D =$ $B + 8$

   $B + B + 4 + B + 8 = 180$

   The measure of the angles are as follows:

   $\angle B =$ _____
   $\angle A =$ _____ } 180°
   $\angle C =$ _____

Now try the following representation:

measure of $\angle B =$ $C - 4$
measure of $\angle C =$ C
measure of $\angle D =$ $C + 4$

$$C - 4 + C + C + 4 = 180$$

The measure of the angles are as follows:

$\angle B =$ _____
$\angle C =$ _____
$\angle D =$ _____
} 180°

Which representation do you prefer? Why? _____

_____

_____

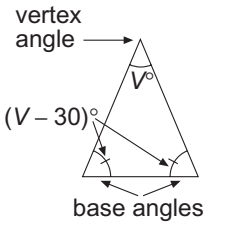
_____

_____

3. The **vertex angle** in an *isosceles triangle* measures 30 degrees more than each of the congruent (≅) **base (*b*)** angles. What is the measure of the *vertex angle*?

Remember: The vertex is opposite to and farthest from the *base* of a triangle. The vertex angle is the angle associated with a given vertex. The *base angles of an isosceles triangle* are congruent.

measure of vertex angle = V
measure of first base angle = $V - 30$
measure of second base angle = $V - 30$



isosceles triangle—has
2 congruent angles

Write an equation and solve the problem.
Show your work.

measure of vertex angle = _____

Check your work. The measure of the angles are as follows:

vertex angle = V = _____
first base angle = $V - 30$ = _____
second base angle = $V - 30$ = _____
} 180°

4. The measures of three angles in a triangle are consecutive multiples of 5. Find the measure of the *largest angle*. Show how you would represent the angle measures. Write an equation and solve. Show your work.

measure of largest angle = _____

The measures of the angles are as follows:

first angle = _____
second angle = _____
third angle = _____
} 180°

5. After solving problem 4, a student said she could have divided 180 by 3 to get the measure of one of the angles and simply *added 5* to get the measure *larger angle* and *subtracted 5* to get the measure of the *smaller angle*. Test her conjecture on problem 2. Show your work. Report your findings.

Findings: _____

_____

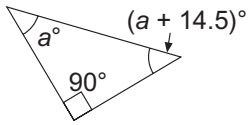
_____

_____

_____

6. In a right triangle, one of the acute angles is 14.5 degrees more than the other. What are the measures of *two acute* angles?

Remember: In a right triangle, one angle is a right angle and two are acute angles.



right triangle—has 1 right angle (90°)

measure of right angle = 90
measure of one acute angle = a
measure of second acute angle = $a + 14.5$

$90 + a + a + 14.5 = 180$

The measures of the angles are as follows:

right angle = 90°
one acute angle = _____
second acute angle = _____
} 180°

7. In a scalene triangle, the measures of the angles are consecutive multiples of 12. What is the measure of the *largest angle*? Show work *or* explain how you got your answer.

measure of largest angle = _____

Explanation: _____

_____

_____

_____

8. In an isosceles triangle, the measure of the *vertex angle* exceeds the measures of each of the *base angles* by 12 degrees. What is the measure of the vertex angle? Show work *or* explain how you got your answer.

measure of the vertex angle = _____

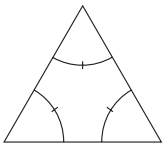
Explanation: _____

_____

_____

_____

9. In an *equilateral triangle*, the measures of *all* angles are congruent. What is the measure of each angle? Show work *or* explain how you got your answer.



equilateral triangle—has 3 congruent angles

measure of each angle = _____

Explanation: _____

_____

_____

_____

10. In a right triangle, the measure of one acute angle is 20 degrees less than the measure of the other acute angle. What is the measure of the *smaller acute angle*? Show your work.

measure of smaller acute angle = _____

Practice

Match each definition with the correct term. Write the letter on the line provided.

_____ 1. a polygon with three sides

_____ 2. the edge of a polygon

_____ 3. a portion of a line that consists of two defined endpoints and all the points in between

_____ 4. a closed-plane figure having at least three sides that are line segments and are connected at their endpoints

_____ 5. the point common to the two rays that form an angle

_____ 6. a one-dimensional measure that is the measurable property of line segments

_____ 7. a portion of a line that begins at an endpoint and goes on indefinitely in one direction

A. length (l)

B. line segment (—)

C. polygon

D. ray (→)

E. side

F. triangle

G. vertex

Practice

Use the list below to write the correct term for each definition on the line provided.

angle (∠)	**consecutive**	**multiples**
base (*b*)	**degree (°)**	**vertex angle**
congruent (≅)	**measure (m) of an angle (∠)**	

_____ 1. the line or plane of a geometric figure, from which an altitude can be constructed, upon which a figure is thought to rest

_____ 2. common unit used in measuring angles

_____ 3. the point about which an angle is measured; the angle associated with a given vertex

_____ 4. the numbers that result from multiplying a given whole number by the set of whole numbers

_____ 5. two rays extending from a common endpoint called the vertex

_____ 6. the number of degrees (°) of an angle

_____ 7. in order

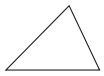
_____ 8. figures or objects that are the same shape and size

Practice

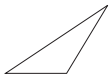
Match each **figure** *with the correct term.*

_____ 1.  has one 90° angle

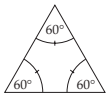
A. acute triangle

_____ 2.  all angles are less than 90° greater than 0°

B. equiangular triangle

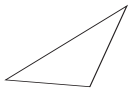
_____ 3.  has one angle more than 90° but less than 180°

C. obtuse triangle

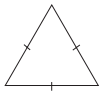
_____ 4. 

all angles are equal

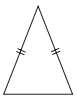
D. right triangle

_____ 5.  has no congruent sides

A. equilateral triangle

_____ 6.  has 3 congruent sides

B. isosceles triangle

_____ 7.  has 2 congruent sides

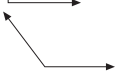
C. scalene triangle

_____ 8.  < 90° and > 0°

A. acute angle

_____ 9.  = 90°

B. obtuse angle

_____ 10.  > 90° and < 180°

C. right angle