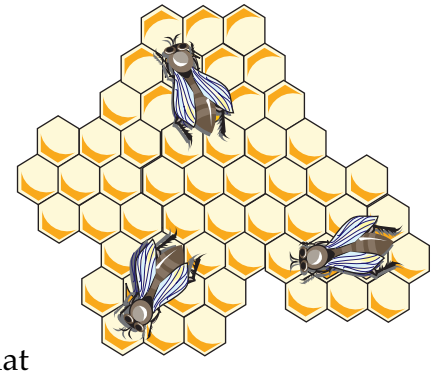
Lesson Three Purpose

- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand and use the real number system. (MA.A.2.4.2)
- Select and justify alternative strategies, such as using properties of numbers, including inverse, identity, distributive, associative, and transitive, that allow operational shortcuts for computational procedures in real-world or mathematical problems. (MA.A.3.4.2)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)
- Use concrete and graphic models to derive formulas for finding rate, distance, time, and angle measurements. (MA.B.1.4.2)
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)
- Describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, tables, and graphs. (MA.D.1.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)

Tessellations



Look at the honey bees in their beehive. Each **hexagon** shape in the beehive is a cell in which the bees store honey. This pattern of *hexagons* is an example of a **tessellation**.

A *tessellation* is a covering or tiling of a *plane* by congruent copies of the same pattern. The shape must be repeated and fit together so that there are no holes and no overlaps between shapes.

In the beehive above, each cell is a *regular hexagon*. A **regular polygon** is a polygon with *all sides congruent* and *all angles congruent*. It is both *equilateral* and *equiangular*. A *regular polygon* with six sides is a *regular hexagon*.

A *regular tessellation* uses congruent regular polygons of only one kind. There are three regular polygons that tessellate: equilateral triangles, **squares**, and hexagons.

All the vertices of the figures fit right next to each other around a point. The sums of the angles around any one point equal 360 degrees.

Think about This!

Since the measure of each angle in a *regular* triangle (equilateral) is 60 degrees and since 60 is a factor of 360, six regular triangles can share a common vertex.

regular equilateral triangles

Since the measure of each angle in a *regular* **quadrilateral**, a *square*, is 90 degrees, and since 90 is a factor of 360, four *regular quadrilaterals* can share a common vertex.

regular quadrilaterals—squares

Since the measure of each angle in a *regular hexagon* is 120 degrees, and since 120 is a factor of 360, three *regular hexagons* can share a common vertex.

regular hexagons

Practice

Solve the following.

Is it possible to use a combination of regular polygons to cover a plane surface and have no holes and no overlaps between the shapes? Consider the following.

1. Four regular polygons have the same side lengths. They meet at a common vertex with no holes and no overlaps. Only two of the polygons are squares. Each angle in one of the other two polygons is 60 degrees more than each angle in the other.

   Show how the measures can be represented symbolically *and* write an equation to determine the measures of the polygons that are not squares.

   measure of each angle in first square = 90

   measure of each angle in second square = 90

   measure of each angle in third polygon = _____

   measure of each angle in fourth polygon = _____

   90 + 90 + _____ + _____ = 360

   Symbolic representation

   Equation: _____

   Explanation: _____

   _____

   _____

   _____

2. Three regular polygons having the same side lengths are placed around a point so that they share a common vertex with no holes and no overlaps. One of the three polygons is a square. The remaining two polygons are congruent. Find the measures of *each angle* sharing this common vertex. Show your work *or* explain how you got your answer.

measure of angle in square = 90
measure of angle in first of two congruent polygons = x
measure of angle in second of two congruent polygons = x

The measures of the angles are as follows:

angle in the square = _____
first of two congruent polygons = _____
second of two congruent polygons = _____
} 360°

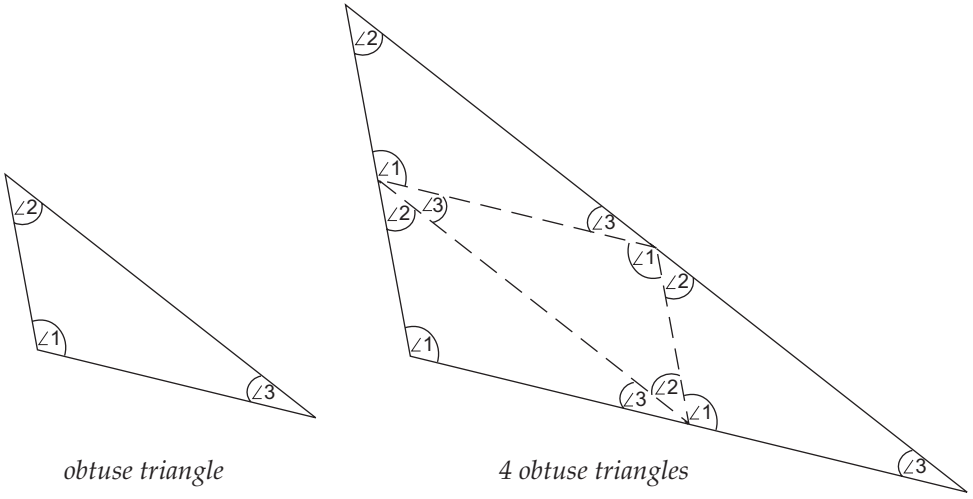
Explanation: _____

_____

_____

_____

3. Four congruent obtuse triangles are placed together as shown in the illustration below to form a larger, **similar** triangle.



obtuse triangle *4 obtuse triangles*

This illustration helps us to see the following:

- **corresponding angles** are equal
- **corresponding sides** are in **proportion**.

The measure of the obtuse angle is twice the sum of the measures of the two acute angles. The measure of one of the acute angles is 10 degrees more than the other. Find the measure of *each angle* in the triangle. Show your work *or* explain how you got your answer.

measure of smaller acute angle = a
measure of larger acute angle = $a + 10$
measure of obtuse angle = $2(a + a + 10)$

The measures of the angles are as follows:

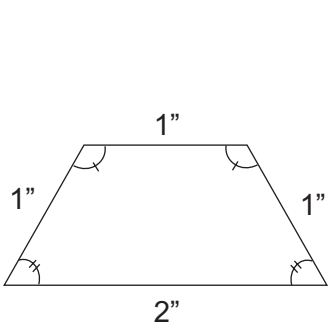
smaller acute angle = _____
larger acute angle = _____ } 180°
obtuse angle = _____

Explanation: _____

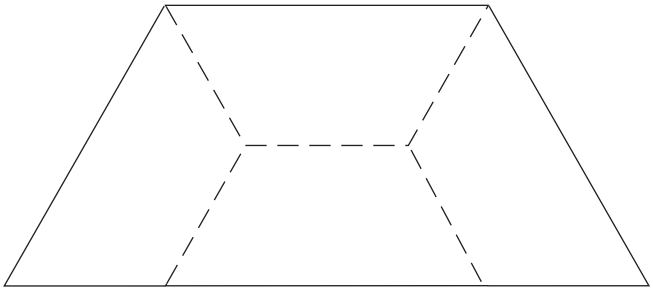
_____

_____

4. Four congruent **isosceles trapezoids** are placed together as shown in the illustration below to form a larger, *similar* isosceles trapezoid.



isosceles trapezoid



4 congruent isosceles trapezoids

Each side of the larger **trapezoid** is twice the length of its *corresponding side* in the smaller *trapezoid*. *Corresponding angles* are congruent. Since this is an *isosceles trapezoid*, the two acute angles are congruent and the two obtuse angles are congruent.

If the measure of an obtuse angle is twice that of an acute angle in the trapezoid, what are the measures of the *angles*? Show your work *or* explain how you got your answer.

The measures of the angles are as follows:

one obtuse angle = _____
other obtuse angle = _____
one acute angle = _____
other acute angle = _____
} 360°

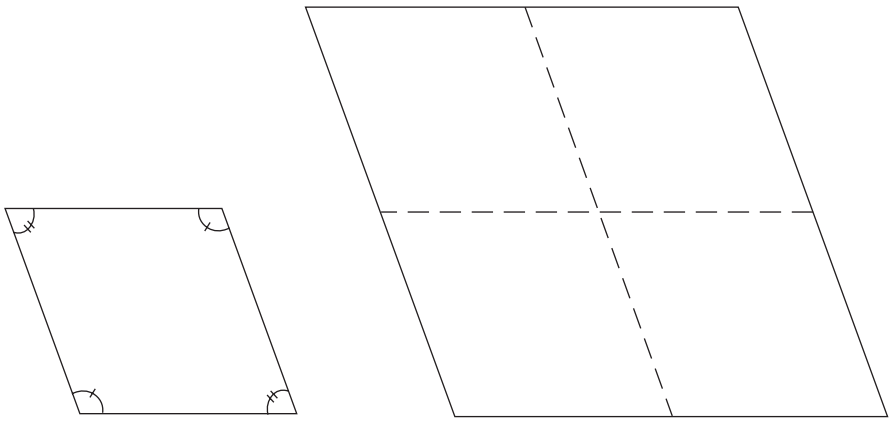
Explanation: _____

_____

_____

_____

5. Four congruent *rhombi* (the plural of **rhombus**) are placed together as shown in the illustration below to form a larger similar *rhombus*.



rhombus *4 congruent rhombi*

Each side length of the larger rhombus is twice its corresponding side length in the smaller rhombus and corresponding angles are congruent. As is true in all **parallelograms**, opposite angles are congruent.

If the measure of each of the two acute angles is one-half the measure of each of the two obtuse angles, what is the measure of *one of the acute angles*? Show your work *or* explain how you got your answer.

measure of one of the acute angles = _____

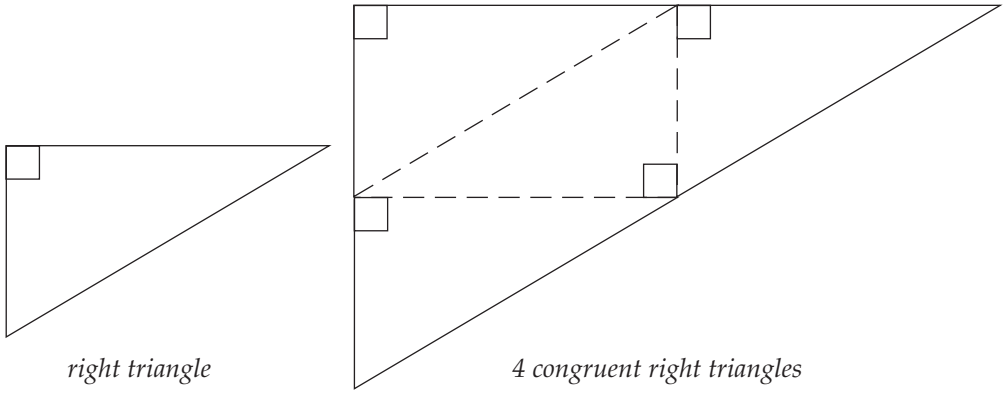
Explanation: _____

_____

_____

_____

6. Four congruent right triangles have been placed together as shown in the illustration below to form a larger, similar right triangle.



right triangle *4 congruent right triangles*

Each side of the larger triangle is twice as long as its corresponding side in the small triangle and the corresponding angles are congruent.

If the measure of one of the acute angles is 20 degrees greater than the other acute angle, what is the measure of the *smaller acute angle*? Show your work *or* explain how you got your answer.

measure of smaller acute angle = _____

Explanation: _____

_____

_____

_____

Practice

Use the list below to complete the following statements.

corresponding angles	**regular polygon**
hexagon	**similar figures**
quadrilateral	**tessellation**

1. _____ are congruent in two similar triangles—thus they have the same measure.

2. A covering or tiling of a plane by congruent copies of the same pattern is a _____ .

3. The measure of each angle in a regular _____ , or square, is 90 degrees.

4. A polygon with six sides is a _____ .

5. Figures that have the same shape; have corresponding, congruent angles; and have corresponding sides that are proportional in length are _____ .

6. A _____ is a polygon with both *equilateral* (all sides congruent) and *equiangular* (all angles congruent).

Practice

Match each definition with the correct term. Write the letter on the line provided.

_____ 1. a quadrilateral with just one pair of opposite sides parallel

_____ 2. a rectangle with four sides the same length

_____ 3. a trapezoid with congruent legs and two pairs of congruent base angles

_____ 4. a parallelogram with four congruent sides

_____ 5. the matching angles and sides in similar figures

_____ 6. a quadrilateral with two pairs of parallel sides

_____ 7. a mathematical sentence stating that two ratios are equal

A. corresponding angles and sides

B. isosceles trapezoid

C. parallelogram

D. proportion

E. rhombus

F. square

G. trapezoid