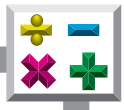


Lesson Four Purpose

- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand and use the real number system. (MA.A.2.4.2)
- Select and justify alternative strategies, such as using properties of numbers, including inverse, identity, distributive, associative, and transitive, that allow operational shortcuts for computational procedures in real-world or mathematical problems. (MA.A.3.4.2)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)
- Use concrete and graphic models to derive formulas for finding rate, distance, time, and angle measurements. (MA.B.1.4.2)
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)
- Describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, tables, and graphs. (MA.D.1.4.1)
- Represent real-world problem situations using finite graphs. (MA.D.2.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)

Equations for Special Angles

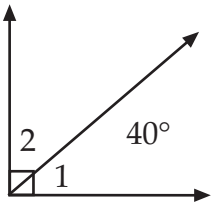
You will use your prior knowledge of special pairs of angles in this lesson. These special pairs include the following:

- **complementary angles**
- **supplementary angles**
- **vertical angles**
- **alternate interior angles**
- **corresponding angles**.

Complementary Angles

Two angles are *complementary angles* if the sum of their measures is 90 degrees.

Example: Study the following picture.



Angle 1 and angle 2 are *complementary angles* because the sum of their measures is 90 degrees. If angle 1 measures 40 degrees, then angle 2 would have to be 50 degrees, because 40 + 50 = 90.

Example: Let's consider the following situation.

In a pair of complementary angles, the measure of one is 25 degrees greater than the measure of the other. Find their measures.

measure of first angle = x
measure of second angle = $x + 25$

$$\begin{aligned} x + x + 25 &= 90 \\ 2x + 25 &= 90 \\ 2x + 25 + (-25) &= 90 + (-25) \\ 2x &= 65 \\ x &= 32.5 \end{aligned}$$

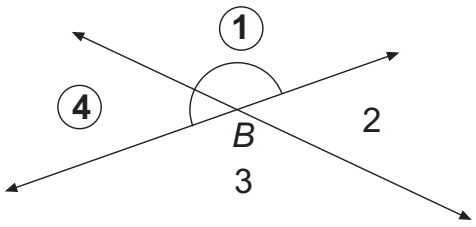
The measure of one of the angles is 32.5 degrees and the measure of the other angle is 25 more than that (32.5 + 25), 57.5 degrees.

measure of first angle = x = 32.5
measure of second angle = $x + 25$ = 57.5
} 90°

Supplementary Angles

Two angles are said to be *supplementary angles* if the sum of their measures is 180 degrees.

Example: Study the following picture.



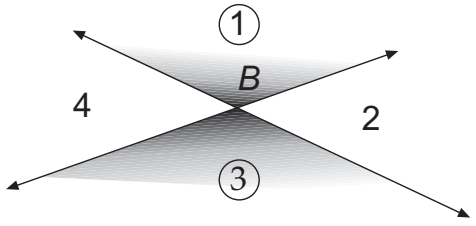
supplementary angles

Angle 1 and angle 4 together make a straight angle. Therefore angle 1 and angle 4 are *supplementary angles*.

Vertical Angles

When two lines intersect, angles that are *opposite* or directly across from each other are called *vertical angles*. Vertical angles are always congruent.

Example: Study the following picture.



vertical angles

Therefore, angle 1 and angle 3 are vertical angles and congruent, and angle 2 and angle 4 are vertical angles and congruent.

Practice

Solve the following. For each problem, show your work **or** *explain how you got your answer.*

1. In a pair of *supplementary angles*, the measure of one is 65 degrees greater than the measure of the other. Find their measures.

Explanation: _____

_____

_____

_____

2. In a pair of *supplementary angles*, the measure of one is 10 degrees less than three times the other. Find their measures.

Explanation: _____

_____

_____

_____

3. In a pair of *complementary angles*, the measure of one is 14 degrees more than twice the other. Find their measures.

Explanation: _____

_____

_____

_____

4. The measures of two *complementary angles* are consecutive multiples of 6. Find their measures.

Explanation: _____

_____

_____

_____

5. The measures of two *supplementary angles* are consecutive **odd numbers**. Find their measures.

Explanation: _____

_____

_____

_____

6. The measures of two *complementary angles* are consecutive **even numbers**. Find their measures.

Explanation: _____

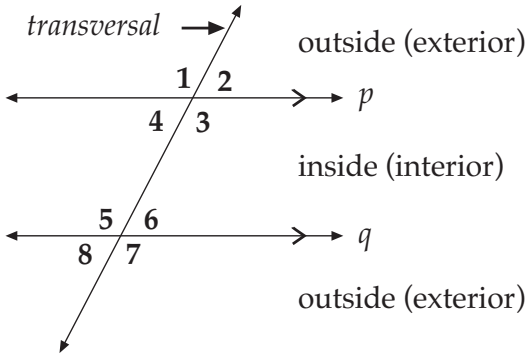
_____

_____

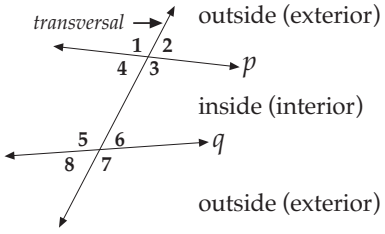
_____

Angles Formed by a Transversal

When a **transversal** intersects two or more lines at different points, it forms eight angles. These angles are **alternate angles**. *Alternate angles* lie on opposite sides and at opposite ends of a *transversal*. The four angles lying between or *inside* the two lines are *alternate interior angles*. The four angles lying *outside* the two lines are *alternate exterior angles*.



Note: Even when lines cut by a transversal are *not* parallel, we still use the same vocabulary.



However, there are special properties when the lines intersected by a transversal are **parallel**.

- ∠1 and 3 are examples of vertical angles.
- ∠5 and 6 are examples of supplementary angles.
- ∠4 and 6 as well as ∠3 and 5 are examples of pairs of *alternate interior angles*. They are between or *inside* the two *parallel lines* on opposite sides of the transversal.
- ∠1 and 7 as well as ∠2 and 8 are examples of pairs of *alternate exterior angles*. They are above and below or *outside* the two *parallel lines* on opposite sides of the transversal.

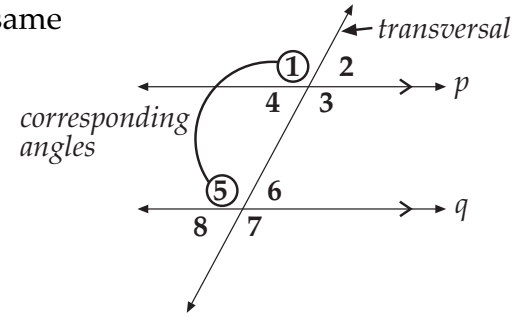
Remember that we have shown that when a transversal intersects two **parallel lines**, the following occurs.

m∠1 = m∠3 vertical angles are congruent
m∠3 = m∠5 alternative angles are congruent
m∠5 = m∠7 vertical angles are congruent

m∠2 = m∠4 vertical angles are congruent
m∠4 = m∠6 alternative angles are congruent
m∠6 = m∠8 vertical angles are congruent

Corresponding Angles

When parallel lines are cut by a transversal, angles in the same relative or matching position are called *corresponding angles*. Corresponding angles also lie on the same side of a transversal.



Therefore,

∠1 corresponds to ∠5
∠6 corresponds to ∠2
∠7 corresponds to ∠3
∠8 corresponds to ∠4.

The measures of the corresponding angles are equal.

Practice

Circle the letter of the correct answer.

1. Angles that are opposite or non-adjacent angles that form when two lines intersect are called _____ .

   a. vertical angles
   b. alternate angles
   c. transversals

2. Two angles are _____ if the sum of their measures is 90 degrees.

   a. supplementary angles
   b. complementary angles
   c. parallel lines

3. Any whole number *not* divisible by two is a(n) _____ .

   a. degree
   b. odd number
   c. even number

4. Any whole number divisible by two is a(n) _____ .

   a. even number
   b. sum
   c. odd number

5. Two angles are said to be _____ if the sum of their measures is 180 degrees.

   a. complementary angles
   b. vertical angles
   c. supplementary angles

6. When a(n) _____ intersects two or more lines at different points, it forms eight angles.

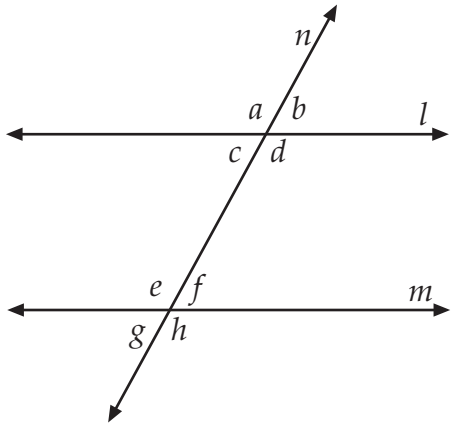
   a. transversal
   b. complementary angle
   c. alternate angle

7. _____ lie on opposite sides and opposite ends of a transversal.

   a. odd numbers
   b. alternate angles
   c. supplementary angles

Practice

Solve the following. Use the **illustration** *below for numbers 1-7.*



1. Lines *l* and *m* are parallel and are cut by a *transversal,* line *n*. If the measure of angle *a* is 30 degrees more than the measure of angle *b*, find the measures of angles *a* and *b*. Show your work *or* explain how you got your answer.

Explanation: _____

_____

_____

_____

Using the **measures** *found in number 1 for* **angles *a* and *b***, *complete the following.*

2. The measure of angle *c* is _____ because

   _____

   _____

   _____

   _____ .

3. The measure of angle *d* is _____ because

   _____

   _____

   _____

   _____ .

4. The measure of angle *e* is _____ because

   _____

   _____

   _____

   _____ .

5. The measure of angle *f* is _____ because

_____

_____

_____

_____ .

6. The measure of angle *g* is _____ because

_____

_____

_____

_____ .

7. The measure of angle *h* is _____ because

_____

_____

_____

_____ .

Practice

Solve the following. Show your work.

Part A

As you find a **value** for x that makes the equation true, think about more than one way to do it. Two ways are shown in the example. You may find other ways.

Example: $3x - 18 = 42$

Method One

$$3x - 18 = 42$$
$$3x - 18 + 18 = 42 + 18$$
$$\frac{3x}{3} = \frac{60}{3}$$
$$x = 20$$

Method Two

From what do I subtract 18 to get 42?
60
Three times what is 60?
20

1. $5x + 24 = 49$

2. $3x - 13 = 35$

3. $7x = 91$

4. $0.5x = 2$

5. $2x + 17 = 24$

6. $-4x + 17 = 57$

7. $2x - (-4) = 20$

Remember: You may rewrite subtraction as addition.

$2x - (-4) = 20$
$2x + 4 = 20$

8. $15x - 26 = -56$

9. $2.5x - (-2) = 12$

10. $13x + 27.5 = 79.5$

Part B

Solve the following. Show all your work.

1. $x + x + 2 + x + 4 = 108$

2. $x - 1 + x + x + 1 = -147$

3. $2x + x + x + 20 = 180$

4. $x + x + 6 + x + 12 + x + 18 = 404$

5. $3x + 2x + x = 180$

6. $6x = 135$

7. $2(x + x - 2) = 36$

8. $2(x + x + 9) = 154$

Match each **expression** *with the correct statement from which the* **expression could have been written**. *Write the letter on the line provided.*

A. $6x$

B. $x - 1 + x + x + 1$

C. $2x + x + x + 20$

D. $x + x + 6 + x + 12 + x + 18$

E. $3x + 2x + x$

F. $x + x + 2 + x + 4$

G. $2(x + x - 2)$

H. $x + x + 8$

_____ 9. The **perimeter** or distance around a regular hexagon.

_____ 10. The *perimeter* of a scalene triangle with side lengths represented by consecutive even **integers**.

_____ 11. The perimeter of a **rectangle** with length and width represented by consecutive even *integers*.

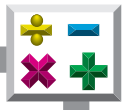
_____ 12. The measures of the angles in a triangle if one angle is twice the smallest and the third angle is 20 more than the smallest.

_____ 13. The scores of four students of golf if the low scorer of the day beat the other players by 6, 12, and 18 strokes.

_____ 14. The ages of two brothers if one is 8 years older than the other.

_____ 15. The measure of the angle of a triangle measures half as much as the second angle and one-third as much as the third angle.

_____ 16. The sum of three consecutive numbers.



Practice

Match each definition with the correct term. Write the letter on the line provided.

_____ 1. the line or plane of a geometric figure, from which an altitude can be constructed, upon which a figure is thought to rest

_____ 2. the order in which any two numbers are added or multiplied does *not* change their sum or product, respectively

_____ 3. a triangle with at least two congruent sides and two congruent angles

_____ 4. the point about which an angle is measured; the angle associated with a given vertex

_____ 5. multiplying the same number on each side of an equation results in an equivalent equation; for any real numbers a, b, and c, if $a = b$, then $ac = bc$

_____ 6. two rays extending from a common endpoint called the vertex

_____ 7. a mathematical sentence in which two expressions are connected by an equality symbol

_____ 8. adding the same number to each side of an equation results in an equivalent equation; for any real numbers a, b, and c, if $a = b$, then $a + c = b + c$

A. addition property of equality

B. angle ($\angle$)

C. base (b)

D. commutative property

E. equation

F. isosceles triangle

G. multiplication property of equality

H. vertex angle

Practice

Use the list below to write the correct term for each definition on the line provided.

alternate angles	**supplementary angles**
corresponding angles	**transversal**
parallel lines	**vertical angles**

_____ 1. a pair of angles that are in matching positions and lie on the same side of a transversal

_____ 2. the opposite or non-adjacent angles formed when two lines intersect

_____ 3. a pair of angles that lie on opposite sides and at opposite ends of a transversal

_____ 4. a line that intersects two or more lines at different points

_____ 5. two lines in the same plane that never meet; also, lines with equal slopes

_____ 6. two angles, with measures the sum of which is exactly 180°