Unit 6: Algebra Applications

This unit emphasizes the use of algebra to solve real-world problems.

Unit Focus

Number Sense, Concepts, and Operations

- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand that numbers can be represented in a variety of equivalent forms, including integers, fractions, decimals, percents, scientific notation, exponents radicals, and absolute value. (MA.A.1.4.4)
- Understand and use the real number system. (MA.A.2.4.2)
- Understand and explain the effects of addition, subtraction, multiplication, and division on real numbers, including square roots, exponents, and appropriate inverse relationships. (MA.A.3.4.1)
- Select and justify alternative strategies, such as using properties of numbers, including inverse, identity, distributive, associative, and transitive, that allow operational shortcuts for computational procedures in real-world or mathematical problems. (MA.A.3.4.2)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)

Measurement

- Solve real-world problems involving rated measures (miles per hour, feet per second). (MA.B.2.4.2)
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)

Algebraic Thinking

- Represent real-world problem situations using finite graphs. (MA.D.2.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)

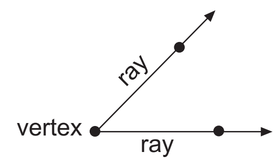
Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

addend .... any number being added
Example: In 14 + 6 = 20,
14 and 6 are addends.

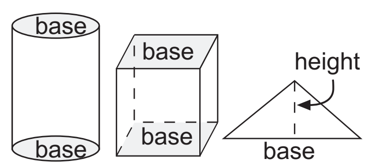
additive identity .... the number zero (0); when zero (0) is added to another number the sum is the number itself
Example: 5 + 0 = 5

angle (∠) .... two rays extending from a common endpoint called the vertex; measures of angles are described in degrees (°)

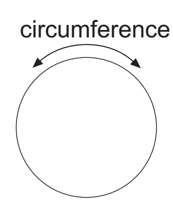


area (*A*) .... the measure, in square units, of the inside region of a two-dimensional figure
Example: A rectangle with sides of 4 units by 6 units contains 24 square units or has an area of 24 square units.

base (*b*) (geometric) .... the line or plane of a geometric figure, from which an altitude can be constructed, upon which a figure is thought to rest

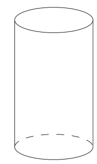


circumference (*C*) .... the distance around a circle



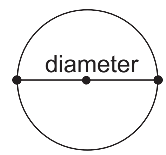
commutative property .... the order in which two numbers are added or multiplied does *not* change their sum or product, respectively
Example: 2 + 3 = 3 + 2 or 4 x 7 = 7 x 4

cylinder .... a three-dimensional figure with two parallel bases that are congruent circles
Example: a can



degree (°) .... common unit used in measuring angles

diameter (*d*) .... a line segment from any point on the circle passing through the center to another point on the circle



divisor .... the number by which another number is divided
Example: In $7\overline{)42}$, 42 ÷ 7, $\frac{42}{7}$, 7 is the divisor.

equivalent
(forms of a number)...................... the same number expressed in different forms
Example: $\frac{3}{4}$, 0.75, and 75%

estimation .... the use of rounding and/or other strategies to determine a reasonably accurate approximation, without calculating an exact answer
Examples: clustering, front-end estimating, and grouping

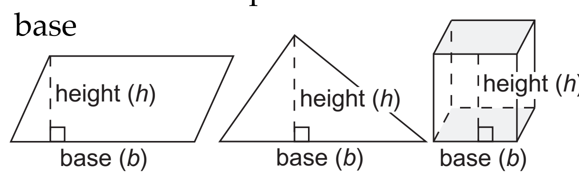
exponent (exponential form) ...... the number of times the base occurs as a factor
Example: 2^3 is the exponential form of $2 \times 2 \times 2$. The numeral two (2) is called the *base*, and the numeral three (3) is called the *exponent*.

expression .... a collection of numbers, symbols, and/or operation signs that stands for a number
Example: $4r^2$; $3x + 2y$; $\sqrt{25}$
Expressions do *not* contain equality (=) or inequality (<, >, ≤, ≥, or ≠) symbols.

factor .... a number or expression that divides evenly into another number
Example: 1, 2, 4, 5, 10, and 20 are factors of 20 and $(x + 1)$ is one of the factors of $(x^2 - 1)$.

formula .... a way of expressing a relationship using variables or symbols that represent numbers

height (*h*) .... a line segment extending from the vertex or *apex* (highest point) of a figure to its base and forming a right angle with the base or plane that contains the base



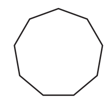
length (*l*) .... a one-dimensional measure that is the measurable property of line segments

like terms .... terms that have the same variables and the same corresponding exponents
Example: In $5x^2 + 3x^2 + 6$, $5x^2$ and $3x^2$ are like terms.

multiplicative identity .... the number one (1); the product of a number and the multiplicative identity is the number itself
Example: 5 x 1 = 5

multiplicative property of zero .... for any number *a*, $a \bullet 0 = 0$ and $0 \bullet a = 0$

nonagon .... a polygon with nine sides



order of operations .... the order of performing computations in parentheses first, then exponents or powers, followed by multiplication and/or division (as read from left to right), then addition and/or subtraction (as read from left to right); also called *algebraic order of operations*
Examples: 5 + (12 – 2) ÷ 2 – 3 x 2 =
5 + 10 ÷ 2 – 3 x 2 =
5 + 5 – 6 =
10 – 6 =
4

perfect square .... a number whose square root is a whole number
Example: 25 is a perfect square because 5 x 5 = 25

perimeter (*P*) .... the distance around a polygon

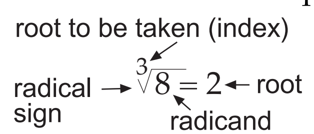
positive numbers .... numbers greater than zero

power (of a number) .... an exponent; the number that tells how many times a number is used as a factor
Example: In 2^3, 3 is the power.

product .... the result of multiplying numbers together
Example: In $6 \times 8 = 48$,
48 is the product.

quotient .... the result of dividing two numbers
Example: In $42 \div 7 = 6$,
6 is the quotient.

radical .... an expression that has a root (square root, cube root, etc.)
Example: $\sqrt{25}$ is a radical
Any root can be specified by an index number, *b*, in the form $\sqrt[b]{a}$ (e.g., $\sqrt[3]{8}$).
A radical without an index number is understood to be a square root.

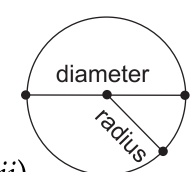


radical

radical sign ($\sqrt{\ }$) .... the symbol ($\sqrt{\ }$) used before a number to show that the number is a *radicand*

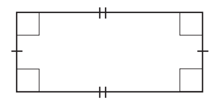
radicand .... the number that appears within a radical sign
Example: In $\sqrt{25}$,
25 is the radicand.

radius (*r*) .... a line segment extending from the center of a circle or sphere to a point on the circle or sphere; (plural: *radii*)



rate/distance .... calculations involving rates, distances, and time intervals, based on the distance, rate, time formula ($d = rt$); a ratio that compares two quantities of different units
Example: feet per second

rectangle .... a parallelogram with four right angles



reflexive property of equality .... a number or expression is equal to itself
Example: $7 = 7$ or $ab = ab$

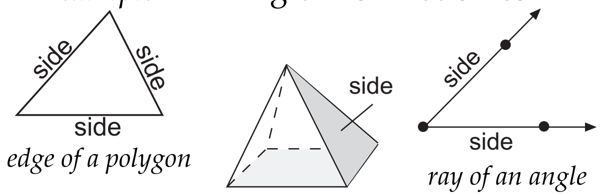
root .... an equal factor of a number
Example:
In $\sqrt{144} = 12$, 12 is the square root.
In $\sqrt[3]{125} = 5$, 5 is the cube root.

rounded number .... a number approximated to a specified place
Example: A commonly used rule to round a number is as follows.

- If the digit in the first place after the specified place is 5 or more, *round up* by adding 1 to the digit in the specified place ($\underline{4}61$ rounded to the nearest hundred is 500).
- If the digit in the first place after the specified place is less than 5, *round down* by *not* changing the digit in the specified place ($\underline{4}41$ rounded to the nearest hundred is 400).

scale factor .... the constant that is multiplied by the lengths of each side of a figure that produces an image that is the same shape as the original figure

side .... the edge of a polygon, the face of a polyhedron, or one of the rays that make up an angle
Example: A triangle has three sides.



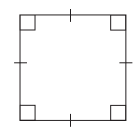
edge of a polygon

face of a polyhedron

ray of an angle

simplify an expression .... to perform as many of the indicated operations as possible

square .... a rectangle with four sides the same length



square (of a number) .... the result when a number is multiplied by itself or used as a factor twice
Example: 25 is the square of 5.

square root (of a number) .... one of two equal factors of a number
Example: 7 is the square root of 49.

square units .... units for measuring area; the measure of the amount of an area that covers a surface

substitution property of equality .... for any numbers *a* and *b*, if $a = b$, then *a* may be replaced by *b*

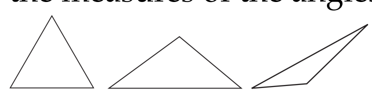
sum .... the result of adding numbers together
Example: In $6 + 8 = 14$, 14 is the sum.

surface area (*S.A.*)
(of a geometric solid) .... the sum of the areas of the faces and any curved surfaces of the figure that create the geometric solid

symmetric property
of equality .... for any numbers a and b, if $a = b$, then $b = a$

transitive
property of equality .... for any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$

triangle .... a polygon with three sides; the sum of the measures of the angles is 180°



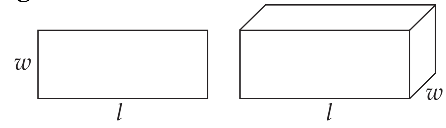
unit .... a precisely fixed quantity used to measure

value (of a variable) .... any of the numbers represented by the variable

variable .... any symbol, usually a letter, which could represent a number

whole number .... the numbers in the set {0, 1, 2, 3, 4, …}

width (*w*) .... a one-dimensional measure of something side to side



Unit 6: Algebra Applications

Introduction

Some of the algebra applications in this chapter will likely be familiar to you and some may be new. Each one will be useful as you study algebra and apply it to the real world.

Lesson One Purpose

- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand that numbers can be represented in a variety of equivalent forms, including integers, fractions, decimals, percents, scientific notation, exponents radicals, and absolute value. (MA.A.1.4.4)
- Understand and use the real number system. (MA.A.2.4.2)
- Understand and explain the effects of addition, subtraction, multiplication, and division on real numbers, including square roots, exponents, and appropriate inverse relationships. (MA.A.3.4.1)
- Select and justify alternative strategies, such as using properties of numbers, including inverse, identity, distributive, associative, and transitive, that allow operational shortcuts for computational procedures in real-world or mathematical problems. (MA.A.3.4.2)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)

- Solve real-world problems involving rated measures (miles per hour, feet per second). (MA.B.2.4.2)
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)
- Represent real-world problem situations using finite graphs. (MA.D.2.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)

Order of Operations

The four problems in the previous practice help us make sense of the appropriate **order of operations** and the need for parentheses at times in moving from words to equations. Although you have previously studied the rules for *order of operations*, here is a quick review.

Rules for Order of Operations

Always start on the *left* and move *to* the *right*.

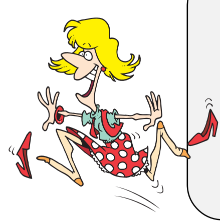
1. Do operations inside *parentheses* first. ( ), [ ], **or** $\frac{x}{y}$
2. Then do all *powers* (*exponents*) **or** *roots*. x^2 **or** $\sqrt{x}$
3. Next do *multiplication* **or** *division*— as they occur from left to right. x **or** ÷
4. Finally, do *addition* **or** *subtraction*— as they occur from left to right. + **or** –

Please note the **or** in *multiplication* **or** *division* and in *addition* **or** *subtraction* in steps 3 and 4.

- This tells you that if multiplication occurs *before* division, do it *first*—as it occurs from left to right.
- If division occurs *before* multiplication, do it *first*—as it occurs from left to right.
- The same is true for addition and subtraction.

The words **as it occurs** *and* from **left** to **right** are very important words.

Sometimes a silly sentence helps to remember the order of operations. Try this one—or make up one of your own.



Please **P**ardon **M**y **D**ear **A**unt **S**ally*

Please .... **P**arentheses
Pardon .... **P**owers
My **D**ear .... **M**ultiplication or **D**ivision
Aunt **S**ally .... **A**ddition or **S**ubtraction

* Also known as **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally—**P**arentheses, **E**xponents, **M**ultiplication or **D**ivision, **A**ddition or **S**ubtraction.

If we are working to **simplify expressions** provided, we apply these rules.

Practice

Use the following information about **algebraic order of operations** *to answer the following statements.*

Study the following.

3 + 5 x 2 = 13	True
3 + 5 x 2 = 16	False
5 + 2 x 2 = 9	True
5 + 2 x 2 = 14	False

1. For the examples marked true, the operation of ________________________ (addition or multiplication) preceded the operation of ________________________ (addition or multiplication).

Study the following.

9 – 4 x 2 = 1	True
9 – 4 x 2 = 10	False
8 – 2 x 3 = 2	True
8 – 2 x 3 = 18	False

2. For the examples marked true, the operation of ________________________ (subtraction or multiplication) preceded the operation of ________________________ (subtraction or multiplication).

Study the following.

12 ÷ 2 x 3 = 18	True
12 ÷ 2 x 3 = 2	False
20 ÷ 4 x 5 = 25	True
20 ÷ 4 x 5 = 1	False

3. For the examples marked true, the operation of

 ________________________ (division or multiplication) preceded the

 operation of ________________________ (division or multiplication).

Study the following.

8 – 3 + 2 = 7	True
8 – 3 + 2 = 3	False

4. For the example marked true, the operation of

 ________________________ (subtraction or addition) preceded the

 operation of ________________________ (subtraction or addition).

Rules and More Rules

Study the following.

$$20 - 4 \times 3 =$$

There are no parentheses. There are no *powers*. We look for multiplication or division and find multiplication. We multiply. We look for addition or subtraction and find subtraction. We subtract.

$$\begin{aligned} 20 - 4 \times 3 &= \\ 20 - 12 &= \\ 8 & \end{aligned}$$

Study the following.

$$8 \div 4 + 8 \div 2 =$$

There are no parentheses. There are no powers. We look for multiplication or division and find division. We divide. We look for addition or subtraction and find addition. We add.

$$\begin{aligned} 8 \div 4 + 8 \div 2 &= \\ 2 + 4 &= \\ 6 & \end{aligned}$$

If the rules were ignored, one might divide 8 by 4 and get 2, then add 2 and 8 to get 10, then divide 10 by 2 to get 5. Agreement is needed.

Study the following.

$$12 - 2^3 =$$

There are no parentheses. We look for powers and find 2^3. We calculate this. We look for multiplication or division and find none. We look for addition or subtraction and find subtraction. We subtract.

$$\begin{aligned} 12 - 2^3 &= \\ 12 - 8 &= \\ 4 & \end{aligned}$$

Study the following.

$$22 - (5 + 2^4) + 7 \times 6 \div 2 =$$

We look for parentheses and find them. We must do what is inside the parentheses first. We find addition and a power. We do the power first and then the addition. We look for multiplication or division and find both. We do them in the order they occur, left to right, and the multiplication occurs first. We look for addition or subtraction and find both. We do them in the order they occur, left to right, so the subtraction occurs first.

$$\begin{aligned}
22 - (5 + 2^4) + 7 \times 6 \div 2 &= \\
22 - (5 + 16) + 7 \times 6 \div 2 &= \\
22 - 21 + 7 \times 6 \div 2 &= \\
22 - 21 + 42 \div 2 &= \\
22 - 21 + 21 &= \\
1 + 21 &= \\
22
\end{aligned}$$

Practice

Apply the **rules for order of operations** *as you solve the following.*

1. My secret number is 4 more than the **product** of 6 and 9. What is my number?

4 more than	the *product* of 6 and 9	is	my secret number
4 +	6 x 9	=	?
4 +	6 x 9	=	________

Think about This!

If you performed the operations as they occurred left to right,

- you would first find the **sum** of 4 and 6.
- You would then find the *product* of 10 and 9.

Your answer would be 90.

If we think about this, 90 is 4 more than 86. We know that 86 is *not* the product of 6 and 9 so our answer is *not* reasonable.

2. My secret number is 13 more than the **square** of the *sum* of 6 and 4. What is my number?

13 more than	the *square* of the sum of 6 and 4	is	my secret number
13 +	$(6 + 4)^2$	=	?
13 +	$(6 + 4)^2$	=	________

Think about This!

If you performed the operations as they occurred left to right,

- you would first find the sum of 13 and 6,
- then the sum of 19 and 4.
- You would then *square* 23, getting 529.

If we think about this, 542 is 13 more than 529. We know that 542 is *not* the square of 10.

3. It is 24 degrees Celsius and I'd like to know the **equivalent** temperature in Fahrenheit. I know it will be 32 more than the product of 1.8 and the Celsius temperature of 24. What is the temperature in Fahrenheit?

32 more than	the product of 1.8 and 24	is	the Fahrenheit *equivalent*
32 +	1.8 x 24	=	?
32 +	1.8 x 24	=	______ °F

Think about This!

If you performed the operations as they occurred left to right,

- you would first find the sum of 32 and 1.8
- you would then multiply 33.8 by 32
- and your answer would be 1,081.6, which is slightly more than 5 times the temperature in Fahrenheit for boiling.

You know this is *not* reasonable.

4. After traveling at an average **rate** of 65 mph for 3 hours, Nick drove 14 more miles. How far did he travel?

14 more than	the distance traveled at 65 mph for 3 hours	is	total distance
14 +	65 x 3	=	?
14 +	65 x 3	=	________ miles

5. The total number of **degrees** in the **angles** of a **nonagon** will be the product of 180 and 2 less than the number of **sides** in the *nonagon* (9). What is the total number of *degrees* in a nonagon?

The product of 180	and 2 less than 9	is	total number of degrees
180 x	(9 – 2)	=	?
180 x	(9 – 2)	=	________ °

6. A **triangle** having a **base (*b*)** of 9 cm and a **height (*h*)** of 8 cm is enlarged by a **scale factor** of 3. Since **area (*A*)** increases by the square of the *scale factor*, the *area* of the enlargement can be determined by finding the product of the area of the original *triangle* and the square of 3. What is the *area* of the enlargement?

The product of the original area	and	the square of 3	is	the area of enlargement
$(\frac{1}{2} \times 9 \times 8)$	+	3^2	=	?
$(\frac{1}{2} \times 9 \times 8)$	+	3^2	=	________

7. To find the **surface area (*S.A.*)** of a **cylinder** we must find the sum of the areas of the two circular *bases* and of the **rectangle** having a **length (*l*)** equal to the **circumference (*C*)** of one of the bases and the *height* of the *cylinder*.

The **formula** can be expressed as the following:

$3.14\,(r^2) + 3.14(r^2) + (3.14d)h$
where *r* represents **radius**,
d represents **diameter** and
h represents *height* of the *cylinder*.

If the *radius* is 5 and the height is 10, what is the *surface area*?

$3.14 \times 5^2 + 3.14 \times 5^2 + 3.14 \times 10 \times 10 =$ ____________

Practice

Write a **number sentence** *to represent each of the following and simplify.*

1. 2 more than the number of quarts in 4 gallons
   Remember: 4 quarts = 1 gallon

   ____

2. 7 less than the number of inches in 2 yards
   Remember: 36 inches = 1 yard

   ____

3. The product of the number of inches in 5 feet and the square of 3
   Remember: 12 inches = 1 foot

   ____

4. The **quotient** of the number of feet in 2 miles and the 4^{th} power of 2
   Remember: 5,280 feet = 1 mile

   ____

5. The sum of the areas of squares with side measures of 9 inches and 15 inches
   Remember: Area (*A*) = side x side

   ____

Simplify the following.

6. $9 - 4 + 5^2 =$

7. $25 - 15 + 4 - 6 =$

8. $2^3 - 2^5 + 2^2 =$

9. $-16 + 2 \times (-5) =$

10. $(\frac{-24}{3}) + 6 \times 7 =$

11. $(19 - 9)^2 + 10 \times 3^2 =$

Practice

Use the list below to write the correct term for each definition on the line provided.

equivalent (forms of a number)	**order of operations**
exponent (exponential form)	**power (of a number)**
expression	**simplify an expression**

______________________ 1. to perform as many of the indicated operations as possible

______________________ 2. the order of performing computations in parentheses first, then exponents or powers, followed by multiplication and/or division (as read from left to right), then addition and/or subtraction (as read from left to right)

______________________ 3. the number of times the base occurs as a factor

______________________ 4. a collection of numbers, symbols, and/or operation signs that stands for a number

______________________ 5. an exponent; the number that tells how many times a number is used as a factor

______________________ 6. the same number expressed in different forms

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. the line or plane of a geometric figure, from which an altitude can be constructed, upon which a figure is thought to rest		A. area (A)
______ 2. the measure, in square units, of the inside region of a two-dimensional figure		B. base (b)
______ 3. a line segment from any point on the circle passing through the center to another point on the circle		C. circumference (C)
______ 4. the distance around a circle		D. diameter (d)
______ 5. the result of a multiplying numbers together		E. product
______ 6. a line segment extending from the center of a circle or sphere to a point on the circle or sphere		F. quotient
______ 7. the result of adding numbers together		G. radius (r)
______ 8. the constant that is multiplied by the lengths of each side of a figure that produces an image that is the same shape as the original figure		H. scale factor
______ 9. the result when a number is multiplied by itself or used as a factor twice		I. square (of a number)
______ 10. the result of dividing two numbers		J. sum
______ 11. the sum of the areas of the faces and any curved surfaces of the figure that create the geometric solid		K. surface area (*S.A.*) (of a geometric solid)