Lesson Two Purpose

- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand that numbers can be represented in a variety of equivalent forms, including integers, fractions, decimals, percents, scientific notation, exponents radicals, and absolute value. (MA.A.1.4.4)
- Understand and use the real number system. (MA.A.2.4.2)
- Understand and explain the effects of addition, subtraction, multiplication, and division on real numbers, including square roots, exponents, and appropriate inverse relationships. (MA.A.3.4.1)
- Select and justify alternative strategies, such as using properties of numbers, including inverse, identity, distributive, associative, and transitive, that allow operational shortcuts for computational procedures in real-world or mathematical problems. (MA.A.3.4.2)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)

Properties

Recall your days in elementary school when you were learning the *addition facts*.

0 + 0	1 + 0	2 + 0	3 + 0	4 + 0	5 + 0	6 + 0	7 + 0	8 + 0	9 + 0
0 + 1	1 + 1	2 + 1	3 + 1	4 + 1	5 + 1	6 + 1	7 + 1	8 + 1	9 + 1
0 + 2	1 + 2	2 + 2	3 + 2	4 + 2	5 + 2	6 + 2	7 + 2	8 + 2	9 + 2
0 + 3	1 + 3	2 + 3	3 + 3	4 + 3	5 + 3	6 + 3	7 + 3	8 + 3	9 + 3
0 + 4	1 + 4	2 + 4	3 + 4	4 + 4	5 + 4	6 + 4	7 + 4	8 + 4	9 + 4
0 + 5	1 + 5	2 + 5	3 + 5	4 + 5	5 + 5	6 + 5	7 + 5	8 + 5	9 + 5
0 + 6	1 + 6	2 + 6	3 + 6	4 + 6	5 + 6	6 + 6	7 + 6	8 + 6	9 + 6
0 + 7	1 + 7	2 + 7	3 + 7	4 + 7	5 + 7	6 + 7	7 + 7	8 + 7	9 + 7
0 + 8	1 + 8	2 + 8	3 + 8	4 + 8	5 + 8	6 + 8	7 + 8	8 + 8	9 + 8
0 + 9	1 + 9	2 + 9	3 + 9	4 + 9	5 + 9	6 + 9	7 + 9	8 + 9	9 + 9

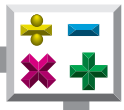
We see 100 addition facts for which 100 flash cards were often made for practice.

Consider, now, the **identity property for addition**:

- The sum of any number and 0 is that same number.
- For any number a, $a + 0 = a$.

This property tells us that the sum of any number and zero is that same number. If we know this property, we don't need flash cards for these facts:

0 + 0	1 + 0	2 + 0	3 + 0	4 + 0	5 + 0	6 + 0	7 + 0	8 + 0	9 + 0



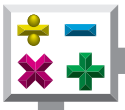
Consider, now, the **commutative property for addition**:

- The order of the **addends** does *not* affect the sum.
- For any numbers a and b, $a + b = b + a$.

1 + 1								
1 + 2	2 + 2							
1 + 3	2 + 3	3 + 3						
1 + 4	2 + 4	3 + 4	4 + 4					
1 + 5	2 + 5	3 + 5	4 + 5	5 + 5				
1 + 6	2 + 6	3 + 6	4 + 6	5 + 6	6 + 6			
1 + 7	2 + 7	3 + 7	4 + 7	5 + 7	6 + 7	7 + 7		
1 + 8	2 + 8	3 + 8	4 + 8	5 + 8	6 + 8	7 + 8	8 + 8	
1 + 9	2 + 9	3 + 9	4 + 9	5 + 9	6 + 9	7 + 9	8 + 9	9 + 9

If we know the sum of 1 + 2, we also know the sum of 2 + 1. One flash card is sufficient.

These two properties allow us to reduce the number of flash cards from 100 to 45 for our addition facts.



Recall your days in elementary school when you were learning the *multiplication facts*.

0 x 0	1 x 0	2 x 0	3 x 0	4 x 0	5 x 0	6 x 0	7 x 0	8 x 0	9 x 0
0 x 1	1 x 1	2 x 1	3 x 1	4 x 1	5 x 1	6 x 1	7 x 1	8 x 1	9 x 1
0 x 2	1 x 2	2 x 2	3 x 2	4 x 2	5 x 2	6 x 2	7 x 2	8 x 2	9 x 2
0 x 3	1 x 3	2 x 3	3 x 3	4 x 3	5 x 3	6 x 3	7 x 3	8 x 3	9 x 3
0 x 4	1 x 4	2 x 4	3 x 4	4 x 4	5 x 4	6 x 4	7 x 4	8 x 4	9 x 4
0 x 5	1 x 5	2 x 5	3 x 5	4 x 5	5 x 5	6 x 5	7 x 5	8 x 5	9 x 5
0 x 6	1 x 6	2 x 6	3 x 6	4 x 6	5 x 6	6 x 6	7 x 6	8 x 6	9 x 6
0 x 7	1 x 7	2 x 7	3 x 7	4 x 7	5 x 7	6 x 7	7 x 7	8 x 7	9 x 7
0 x 8	1 x 8	2 x 8	3 x 8	4 x 8	5 x 8	6 x 8	7 x 8	8 x 8	9 x 8
0 x 9	1 x 9	2 x 9	3 x 9	4 x 9	5 x 9	6 x 9	7 x 9	8 x 9	9 x 9

We see 100 multiplication facts for which 100 flash cards were often made for practice.

Consider, now, the **multiplicative property of zero**:

- The product of any number and zero is zero.
- For any number a, $a(0) = 0$.

If we know this property, we know that any number times zero is zero. We eliminate the first column and the first row of multiplication facts.

Consider, now, the **identity property for multiplication**:

- The product of any number and 1 is that same number.
- For any number a, $a(1) = a$.

If we know this property, we know that any number times one is that same number. We eliminate the second column of multiplication facts as well as the second row.

Consider, now, the **commutative property for multiplication**:

- The numbers can be multiplied in any order.
- For any numbers a and b, $a \bullet b = b \bullet a$

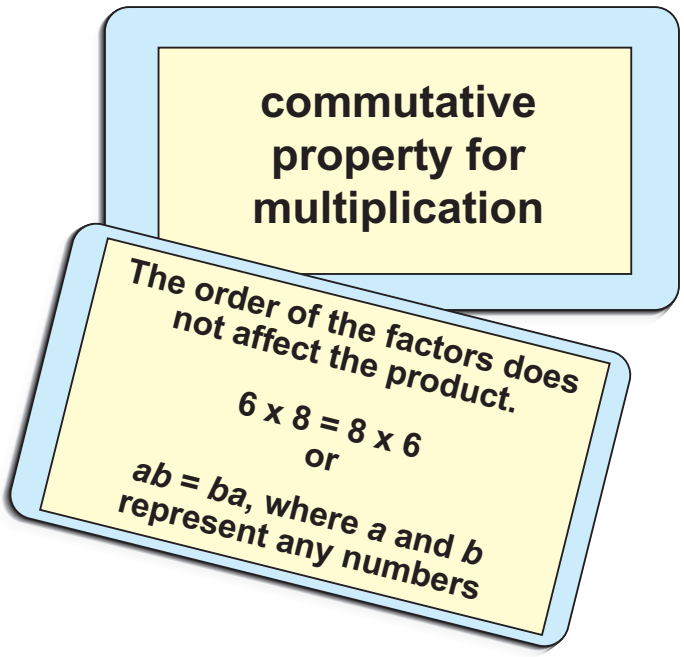
If we know this property, we know that 6 x 9 has the same product as 9 x 6. Only one card will be needed to represent these two facts.

These properties help us reduce the number of facts to be learned as shown in the following table:

2 x 2							
2 x 3	3 x 3						
2 x 4	3 x 4	4 x 4					
2 x 5	3 x 5	4 x 5	5 x 5				
2 x 6	3 x 6	4 x 6	5 x 6	6 x 6			
2 x 7	3 x 7	4 x 7	5 x 7	6 x 7	7 x 7		
2 x 8	3 x 8	4 x 8	5 x 8	6 x 8	7 x 8	8 x 8	
2 x 9	3 x 9	4 x 9	5 x 9	6 x 9	7 x 9	8 x 9	9 x 9

We now have 36 multiplication facts to learn if we know the *multiplicative property of zero*, the identity property for multiplication, and the commutative property for multiplication.

You have probably been using these properties for a long time whether or not you knew the name of the properties. You may find it helpful to make some flash cards for properties. One side of the card should have the name of the property and the other side should have an explanation and example of the property. For example:



Study the chart below.

Order (Commutative Property)	
Commutative Property of Addition: ***a* + *b* = *b* + *a*** Numbers can be added in any order. 10 + 2 = 2 + 10 *x* + 2 = 2 + *x*	**Commutative Property of Multiplication:** ***ab* = *ba*** Numbers can be multiplied in any order. 2 • 10 = 10 • 2 2 • *x* = *x* • 2
Identity Properties	
Additive Identity: 0 is the additive identity. ***a* + 0 = *a* and 0 + *a* = *a*** The sum of any number and zero is the number. 5 + 0 = 5 *x* + 0 = *x*	**Multiplicative Identity: 1 is the multiplicative identity.** ***a* • 1 = *a* and 1 • *a* = *a*** The product of any number and one is the number. 5 • 1 = 5 *x* • 1 = *x*

These properties deal with the following:

order—**commutative property** of addition and commutative property of multiplication

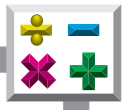
identity—**additive identity** property and **multiplicative identity** property

zero—**multiplicative property of zero**

Properties of Equality

Four additional examples of properties you have likely used, whether or not you knew the names for them, are as follows:

reflexive property of equality
• For any number a, $a = a$. • This is another way of saying that any quantity is equal to itself.
symmetric property of equality
• For any numbers a and b, if $a = b$, then $b = a$. • This is another way of saying if one quantity equals a second, then the second equals the first.
transitive property of equality
• For any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$. • This is another way of saying if one quantity equals a second, and the second quantity equals a third, then the first quantity equals the third.
substitution property of equality
• For any numbers a and b, if $a = b$, then a may be replaced by b in any expression.



Study the chart and examples below describing properties of equality.

Properties of Equality

Reflexive:	$a = a$
Symmetric:	If $a = b$, then $b = a$.
Transitive:	If $a = b$ and $b = c$, then $a = c$.
Substitution:	If $a = b$, then a may be replaced by b.

Examples of Properties of Equality

Reflexive:	$8 - e = 8 - e$
Symmetric:	If $5 + 2 = 7$, then $7 = 5 + 2$.
Transitive:	If $9 - 2 = 4 + 3$ and $4 + 3 = 7$, then $9 - 2 = 7$.
Substitution:	If $x = 8$, then $x \div 4 = 8 \div 4$. x is replaced by 8.
	or
	If $9 + 3 = 12$, then $9 + 3$ may be replaced by 12.

Practice

Match each **property** *with the correct* **example of the property**. *Assume all* **variables represent numbers**. *Write the letter on the line provided.* **Some terms will be used more than once.**

_____ 1. $3 + 4 = 4 + 3$

_____ 2. $4 = 4$

_____ 3. If $3 + 4 = 7$, then $7 = 4 + 3$.

_____ 4. $6(0) = 0$

_____ 5. $32(56) = 56(32)$

_____ 6. If $5 + 2 = 7$ and $7 = 4 + 3$, then $5 + 2 = 4 + 3$.

_____ 7. $6 + 0 = 6$

_____ 8. $72(1) = 72$

_____ 9. $0(a) = a(0)$

_____ 10. $b(1) = b$

_____ 11. $c + d = d + c$

_____ 12. $e(0) = 0$

_____ 13. If $g = h$ and $h = k$, then $g = k$.

_____ 14. $m + 0 = m$

_____ 15. $rs = sr$

_____ 16. If $u = v$, then $v = u$.

A. identity property for addition

B. identity property for multiplication

C. commutative property for addition

D. commutative property for multiplication

E. multiplicative property of zero

F. reflexive property

G. symmetric property

H. transitive property