Lesson Three Purpose

- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand that numbers can be represented in a variety of equivalent forms, including integers, fractions, decimals, percents, scientific notation, exponents radicals, and absolute value. (MA.A.1.4.4)
- Understand and use the real number system. (MA.A.2.4.2)
- Understand and explain the effects of addition, subtraction, multiplication, and division on real numbers, including square roots, exponents, and appropriate inverse relationships. (MA.A.3.4.1)
- Select and justify alternative strategies, such as using properties of numbers, including inverse, identity, distributive, associative, and transitive, that allow operational shortcuts for computational procedures in real-world or mathematical problems. (MA.A.3.4.2)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)
- Solve real-world problems involving rated measures (miles per hour, feet per second). (MA.B.2.4.2)
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)

Finding the Perimeter of Rectangles

You will use the following examples to test the properties.

- A *formula* for finding the **perimeter (*P*)** of a *rectangle* is

  $$2l + 2w = P$$

  where l represents *length*, w represents **width**, and P represents *perimeter*.

- The commutative property for addition tells us that another formula could be

  $$2w + 2l = P.$$

- We know another formula for finding the perimeter of a rectangle is

  $$2(l + w) = P.$$

Practice

Complete the following table by **testing each formula** *to find the* **perimeter of rectangles** *with the dimensions shown.*

Finding the Perimeter of Rectangles

	Values for length and width in centimeters	$2l + 2w = P$	$2w + 2l = P$	$2(l + w) = P$
	$l = 5$, $w = 9$	2(5) + 2(9) = 10 + 18 = 28	2(9) + 2(5) = 18 + 10 = 28	2(5 + 9) = 2(14) = 28
1.	$l = 7$, $w = 4$			
2.	$l = 8.5$, $w = 9.2$			
3.	$l = 2.5$, $w = 1.25$			
4.	$l = 17.6$, $w = 2.4$			
5.	$l = 10$, $w = 0.75$			
6.	$l = 1\frac{1}{8}$, $w = 4\frac{1}{4}$			

Finding Distance

We know the formula,

$$rt = d,$$

where r represents *rate*, t represent time, and d represents *distance*, which allows us to determine distance if we know the rate of speed and the time traveled.

The commutative property for multiplication would also allow the formula to be expressed as

$$tr = d.$$

Practice

Complete the following table by **testing each formula** *for the following* **values of rate and time.**

Finding Distance

	Values for rate in kilometers per hour and time in hours	*rt* = *d*	*tr* = *d*
1.	$r = 100$, $t = 5.75$		
2.	$r = 50$, $t = 2.5$		
3.	$r = 100$, $t = 10.25$		
4.	$r = 40$, $t = 0.5$		
5.	$r = 100$, $t = 5.75$		

Factor Pairs

Another example of the commutative property for multiplication is observed when we are finding all the *factor pairs* for a number. Consider the following:

number	factor pairs
30	1 x 30, 2 x 15, 3 x 10, 5 x 6, **6 x 5**, **10 x 3**, **15 x 2**, **30 x 1**
54	1 x 54, 2 x 27, 3 x 18, 6 x 9, **9 x 6**, **18 x 3**, **27 x 2**, **54 x 1**
18	1 x 18, 2 x 9, 3 x 6, **6 x 3**, **9 x 2**, **18 x 1**
300	1 x 300, 2 x 150, 3 x 100, 4 x 75, 5 x 60, 6 x 50, 10 x 30, 12 x 50, 15 x 20, **20 x 15**, **50 x 12**, **30 x 10**, **50 x 6**, **60 x 5**, **75 x 4**, **100 x 3**, **150 x 2**, **300 x 1**

The commutative property for multiplication tells us that 5 x 6 = 6 x 5. When we are finding factor pairs, we generally don't need to list both of these. In some special cases, it is important to consider both. One such case might be organizing 300 band members into rows with an equal number of people. We know that 10 rows of 30 band members is different from 30 rows of 10 band members. We also know that each arrangement results in a total of 300 band members.

Factoring and Square Roots

Another interesting thing to note is the relationship of the **square root** of a number and factor pairs appearing in reverse order.

- The *square root* of 30, the first number in the table on the previous page, is more than 5 but less than 6. If we test numbers 1-5 to see if they provide factor pairs for 30, we don't need to test further.
- The square root of 54 is more than 7 but less than 8. If we test numbers 1-7 to see if they provide factor pairs for 54, we don't need to test further.
- The square root of 18 is more than 4 but less than 5. If we test numbers 1-4 to see if they provide factor pairs for 18, we don't need to test further.
- The square root of 300 is more than 17 but less than 18. If we test numbers 1-17 to see if they provide factor pairs for 300, we don't need to test further.

Practice

Complete the following table.

Finding Factor Pairs

	number	square root of number	check trial divisors ____ to ____	factor pairs
	40	≈ 6.3	1 to 6	1 x 40, 2 x 20, 4 x 10, 5 x 8
1.	90			
2.	121			
3.	48			
4.	125			
5.	60			
6.	12			
7.	400			
8.	91			

Remember: The ≈ symbol means *is approximately equal to*. The ≈ symbol is used with a number that describes another number without specifying it exactly.

Measuring with Factor Pairs

It is interesting to note that each factor pair for a number can represent the length and *width* of a rectangle having that number of **square units** in its area. For example:

number	factor pairs
54	1 x 54, 2 x 27, 3 x 18, 6 x 9

Using Factor Pairs to Find Measurements of Rectangles

Length of rectangle in units	**Width of rectangle in units**	**Area of rectangle in square units**	**Perimeter of rectangle in units** $P = 2(l + w)$
1	54	54	2(54 + 1) = 110
2	27	54	2(2 + 27) = 58
3	18	54	2(3 + 18) = 42
6	9	54	2(6 + 9) = 30

The long, skinny rectangle (1 by 54), with an area of 54 *square units* has the greatest *perimeter*. The rectangle closest to a square (6 by 9), with an area of 54 square units, has the smallest perimeter.

Practice

Complete the following table.

Using Factor Pairs to Find Measurements of Rectangles

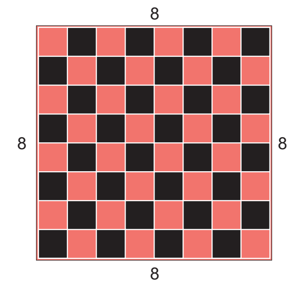
	Length of rectangle in inches	Width of rectangle in inches	Area of rectangle in square inches	Perimeter of rectangle in inches
1.			36	
2.			36	
3.			36	
4.			36	
5.			36	

Complete the following table.

Using Factor Pairs to Find Measurements of Rectangles

	Length of rectangle in inches	Width of rectangle in inches	Area of rectangle in square inches	Perimeter of rectangle in inches
6.	1		900	
7.	2		900	
8.	3		900	
9.	4		900	
10.	5		900	
11.	6		900	
12.	9		900	
13.	10		900	
14.	12		900	
15.	15		900	
16.	18		900	
17.	20		900	
18.	25		900	
19.	30		900	

Working with Radicals



Using Square Roots

A checkerboard is a **perfect square** containing 64 little **squares**. Each side of a *square* has the same length and width. We know that $8^2 = 64$, and each side of the checkerboard is 8 **units**. The opposite of squaring a number is called *finding the square root of a number*. The *square root* of 64 or $\sqrt{64}$ is 8.

The square root of a number is shown by the symbol $\sqrt{}$, which is called a **radical sign** or *square root sign*. The number underneath is called a **radicand**. The **radical** is an *expression* that has a *root*. A root is an equal factor of a number.

radical sign → $\sqrt{100}$ ← radicand

radical

$$\sqrt{100} = 10 \text{ because } 10^2 = 100$$

$$\sqrt{9} = 3 \text{ because } 3^2 = 9$$

$$\sqrt{121} = 11 \text{ because } 11^2 = 121$$

The numbers 100, 9, and 121 are *perfect squares* because their square roots are **whole numbers**. What do we do if the *radicand* is *not* a *perfect square*? We have three options for finding the square root of a number:

Option 1: We can refer to a *square root chart*. Below is a partial table of squares and square roots. See Appendix A for a more complete table.

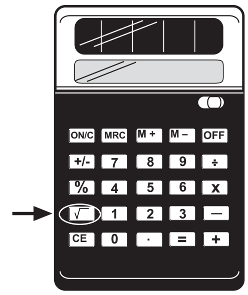
$\sqrt{6} \approx 2.449$

This answer is **rounded** to the nearest thousandth.

Table of Squares and Approximate Square Roots

n	***n*²**	**√*n***
1	1	1.000
2	4	1.414
3	9	1.732
4	16	2.000
5	25	2.236
6	36	2.449
7	49	2.646
8	64	2.828
9	81	3.000
10	100	3.162

Option 2: We can use a *calculator*. Look for a key with the $\sqrt{\ }$ symbol. Enter 6, hit this key, and you will get 2.44948974278. This result is a decimal *approximation* of the $\sqrt{6}$. You will have to *round* the number to the nearest thousandth.



$\sqrt{6} = 2.44\underline{9}48974278$

$\sqrt{6} \approx 2.449$

Option 3: We can **estimate**. We know

$\sqrt{4} = 2$

$\sqrt{6} \approx ?$

$\sqrt{9} = 3$

$\sqrt{6}$ is about half way between $\sqrt{4}$ and $\sqrt{9}$, so a good guess would be 2.5.

Note: Appendix B contains a list of mathematical symbols and their meanings and Appendix C contains formulas and conversions.

Multiplying Radical Expressions

A calculator produced the following products. You might verify these with your own calculator.

$(\sqrt{5})(\sqrt{5}) = 5$

$(\sqrt{21})(\sqrt{21}) = 21$

$(\sqrt{99})(\sqrt{99}) = 99$

$(\sqrt{4})(\sqrt{9}) = 6$

$(\sqrt{3})(\sqrt{12}) = 6$

$(\sqrt{2})(\sqrt{18}) = 6$

$(\sqrt{6})(\sqrt{6}) = 6$

$(\sqrt{2})(\sqrt{50}) = 10$

$(\sqrt{4})(\sqrt{25}) = 10$

$(\sqrt{5})(\sqrt{20}) = 10$

$(\sqrt{10})(\sqrt{10}) = 10$

If you are ready to conjecture that $(\sqrt{a})(\sqrt{b}) = \sqrt{ab}$, your conjecture is true, when a and b are **positive numbers**.

The rule also applies to radical expressions such as the following:

$(\sqrt{15})(\sqrt{3}) = \sqrt{45} = \sqrt{9 \bullet 5} = 3\sqrt{5}$ *or* 6.71

$(\sqrt{6})(\sqrt{90}) = \sqrt{540} = \sqrt{36 \bullet 15} = 6\sqrt{15}$ *or* 23.24

Practice

Express the **product in simplified radical form**.

1. $(\sqrt{18})(\sqrt{5}) =$

2. $(\sqrt{15})(\sqrt{8}) =$

3. $(\sqrt{10})(\sqrt{50}) =$

4. $(\sqrt{56})(\sqrt{10}) =$

5. $(\sqrt{7})(\sqrt{18}) =$

6. $(\sqrt{15})(\sqrt{20}) =$

7. $(\sqrt{50})(\sqrt{75}) =$

8. $(\sqrt{12})(\sqrt{7}) =$

9. $(\sqrt{13})(\sqrt{26}) =$

10. $(\sqrt{11})(\sqrt{15}) =$

More about Radical Expressions

a. $(\sqrt{18})(\sqrt{5}) = \sqrt{90} = \sqrt{9 \bullet 10} = 3\sqrt{10}$

or

$(\sqrt{18})(\sqrt{5}) = \sqrt{9 \bullet 2} \bullet \sqrt{5} = 3\sqrt{2} \bullet \sqrt{5} = 3\sqrt{10}$

b. $\sqrt{50} \bullet \sqrt{75} = \sqrt{3750} = \sqrt{25 \bullet 150} = 5\sqrt{150} = 5\sqrt{25 \bullet 6} = 25\sqrt{6}$

or

$\sqrt{50} \bullet \sqrt{75} = \sqrt{25 \bullet 2} \bullet \sqrt{25 \bullet 3} = 5\sqrt{2} \bullet 5\sqrt{3} = 25\sqrt{6}$

Radical expressions can be simplified before being multiplied or afterwards. The final product will be the same. The choice is yours!

Consider the following sums:

$\sqrt{6} + \sqrt{24} =$

$\sqrt{6} + \sqrt{4 \bullet 6} =$

$\sqrt{6} + 2\sqrt{6} =$ Note the *understood* coefficient of $\sqrt{6}$ is 1
$\sqrt{6} = 1\sqrt{6}$

$3\sqrt{6}$

$$\sqrt{20} + \sqrt{80} =$$
$$\sqrt{4 \bullet 5} + \sqrt{16 \bullet 5} =$$
$$2\sqrt{5} + 4\sqrt{5} =$$
$$6\sqrt{5}$$

$$\sqrt{98} + \sqrt{8} + \sqrt{12} =$$
$$\sqrt{49 \bullet 2} + \sqrt{4 \bullet 2} + \sqrt{4 \bullet 3} =$$
$$7\sqrt{2} + 2\sqrt{2} + 2\sqrt{3} =$$
$$9\sqrt{2} + 2\sqrt{3}$$

Note the difference in **like terms** and *unlike terms*.

$$2\sqrt{40} + 3\sqrt{40} =$$
$$2\sqrt{4 \bullet 10} + 3\sqrt{4 \bullet 10} =$$
$$4\sqrt{10} + 6\sqrt{10} =$$
$$10\sqrt{10}$$

or

$$2\sqrt{40} + 3\sqrt{40} =$$
$$5\sqrt{40} =$$
$$5\sqrt{4 \bullet 10} =$$
$$10\sqrt{10}$$

Think about This!

When adding like radical expressions, simplification can be completed before or after finding the sum.

When adding unlike radical expressions, simplification must be completed before addition so that *like terms* can be combined.

Practice

Express the **sums in simplified radical form**.

1. $2\sqrt{5} + \sqrt{20} =$

2. $3\sqrt{60} + 2\sqrt{15} =$

3. $\sqrt{40} + \sqrt{90} =$

4. $3\sqrt{7} + 2\sqrt{28} + 5\sqrt{14} =$

5. $3\sqrt{27} + 7\sqrt{12} =$

6. $\sqrt{5} + \sqrt{125} =$

7. $\sqrt{1} + \sqrt{4} + \sqrt{9} =$

8. $\sqrt{50} + \sqrt{70} =$

9. $8\sqrt{32} + 5\sqrt{48} =$

10. $\sqrt{54} + \sqrt{24} =$

Practice

Match each **property** *with the correct* **example of the property**. *Assume all* **variables represent numbers**. *Write the letter on the line provided.*

______	1. $a \bullet 0 = 0$ and $0 \bullet a = 0$	A. additive identity
______	2. $a + b = b + a$ or $ab = ba$	B. commutative property
______	3. $a \bullet 1 = a$ and $1 \bullet a = a$	C. multiplicative identity
______	4. if $a = b$ and $b = c$, then $a = c$	D. multiplicative property of zero
______	5. $a + 0 = a$ and $0 + a = a$	E. transitive property

Practice

Use the list below to write the correct term for each definition on the line provided.

divisor	**radical**	**simplify an expression**
factor	**radicand**	**square root (of a number)**
order of operations	**rounded number**	**value (of a number)**
perfect square		

______________________ 1. the order of performing computations in parentheses first, then exponents or powers, followed by multiplication and/or division (as read from left to right), then addition and/or subtraction (as read from left to right)

______________________ 2. the number that appears within a radical sign

______________________ 3. a number or expression that divides evenly into another number

______________________ 4. to perform as many of the indicated operations as possible

______________________ 5. any of the numbers represented by the variable

______________________ 6. a number whose square root is a whole number

______________________ 7. an expression that has a root (square root, cube root, etc.)

______________________ 8. a number approximated to a specified place

______________________ 9. the number by which another number is divided

______________________ 10. one of two equal factors of a number