Unit 7: Discoveries about Linear Relationships

This unit emphasizes linear relationships and the impact of changing parameters of given functions.

Unit Focus

Number Sense, Concepts, and Operations

- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand and use the real number system. (MA.A.2.4.2)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)

Measurement

- Use concrete and graphic models to derive formulas for finding rate, distance, time, and angle measurements. (MA.B.1.4.2)
- Solve real-world problems involving rated measures (miles per hour, feet per second). (MA.B.2.4.2)
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)

Geometry and Spatial Sense

- Using a rectangular coordinate system (graph), apply and algebraically verify properties of two-dimensional figures, including distance, midpoint, slope, parallelism, and perpendicularity. (MA.C.3.4.2)

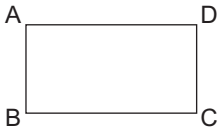
Algebraic Thinking

- Describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, tables, and graphs. (MA.D.1.4.1)
- Determine the impact when changing parameters of given functions. (MA.D.1.4.2)
- Represent real-world problem situations using finite graphs. (MA.D.2.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)

Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

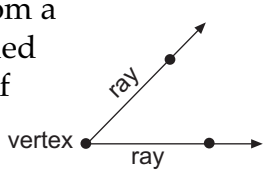
adjacent sides ..... sides that are next to each other and share a common vertex



Sides *AB* and *AD* are adjacent. They share the common vertex *A*.

altitude ..... the perpendicular distance from a vertex in a polygon to its opposite side

angle (∠) ..... two rays extending from a common endpoint called the vertex; measures of angles are described in degrees (°)



axes (of a graph) ..... the horizontal and vertical number lines used in a coordinate plane system; (singular: *axis*)

coefficient ..... a numerical factor in a term of an algebraic expression
Example: In $8b$, the coefficient of b is 8.

congruent (≅) ..... figures or objects that are the same shape and the same size

consecutive ..... in order
Example: 6, 7, 8 are consecutive whole numbers and 4, 6, 8 are consecutive even numbers.

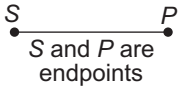
coordinate grid or plane ..... a two-dimensional network of horizontal and vertical lines that are parallel and evenly-spaced; especially designed for locating points, displaying data, or drawing maps

coordinates ..... numbers that correspond to points on a coordinate plane in the form (x, y), or a number that corresponds to a point on a number line

data ..... information in the form of numbers gathered for statistical purposes

difference ..... a number that is the result of subtraction
Example: In $16 - 9 = 7$, 7 is the difference.

endpoint ..... either of two points marking the end of a line segment



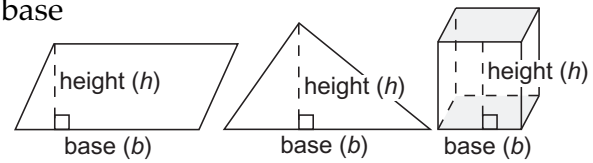
equation ..... a mathematical sentence in which two expressions are connected by an equality symbol
Example: $2x = 10$

equivalent (forms of a number) ..... the same number expressed in different forms
Example: $\frac{3}{4}$, 0.75, and 75%

formula ..... a way of expressing a relationship using variables or symbols that represent numbers

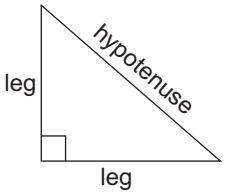
graph ..... a drawing used to represent data
Example: bar graphs, double bar graphs, circle graphs, and line graphs

height (*h*) ..... a line segment extending from the vertex or *apex* (highest point) of a figure to its base and forming a right angle with the base or plane that contains the base



horizontal ..... parallel to or in the same plane of the horizon

hypotenuse ..... the longest side of a right triangle; the side opposite the right angle

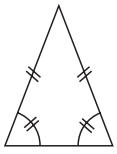


increase ..... to make greater

intersect ..... to meet or cross at one point

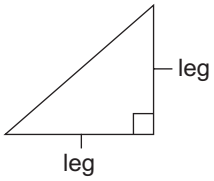
intersection ..... the point at which lines or curves meet; the line where planes meet

isosceles triangle ..... a triangle with two congruent sides and two congruent angles



labels (for a graph) ..... the titles given to a graph, the axes of a graph, or the scales on the axes of a graph

leg ..... in a right triangle, one of the two sides that form the right angle

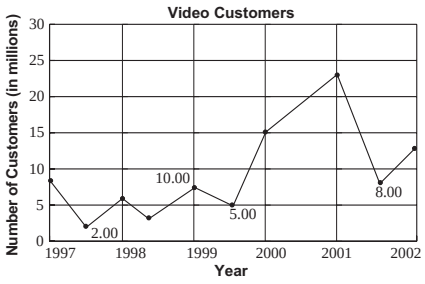


length (l) ..... a one-dimensional measure that is the measurable property of line segments

line ($\longleftrightarrow$) ..... a collection of an infinite number of points in a straight pathway with unlimited length and having no width

linear equation ..... an algebraic equation in which the variable quantity or quantities are raised to the zero or first power and the graph is a straight line
Example: $20 = 2(w + 4) + 2w$; $y = 3x + 4$

line graph ..... a graph that displays data using connected line segments



line segment (—) ..... a portion of a line that consists of two defined endpoints and all the points in between
Example: The line segment AB is between point A and point B and includes point A and point B.

mean (or average) ..... the arithmetic average of a set of numbers; a measure of central tendency

midpoint (of a line segment) ..... the point on a line segment equidistant from the endpoints

minimum ..... the smallest amount or number allowed or possible

negative numbers ..... numbers less than zero

opposite sides ..... sides that are directly across from each other

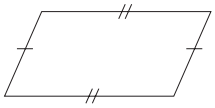
ordered pair ..... the location of a single point on a rectangular coordinate system where the first and second values represent the position relative to the x-axis and y-axis, respectively
Example: (x, y) or $(3, -4)$

origin ..... the point of intersection of the x- and y-axes in a rectangular coordinate system, where the x-coordinate and y-coordinate are both zero (0)

parallel (∥) ..... being an equal distance at every point so as to never intersect

parallel lines ..... two lines in the same plane that are a constant distance apart; lines with equal slopes

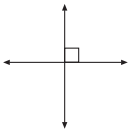
parallelogram ..... a quadrilateral with two pairs of parallel sides



pattern (relationship) ..... a predictable or prescribed sequence of numbers, objects, etc; may be described or presented using manipulatives, tables, graphics (pictures or drawings), or algebraic rules (functions)
Example: 2, 5, 8, 11 … is a pattern. Each number in this sequence is three more than the preceding number. Any number in this sequence can be described by the algebraic rule, $3n - 1$, by using the set of counting numbers for n.

perpendicular (⊥) ..... two lines, two line segments, or two planes that intersect to form a right angle

perpendicular lines ..... two lines that intersect to form right angles

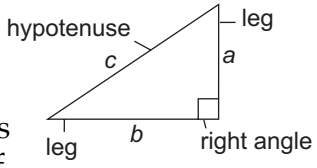


point ..... a specific location in space that has no discernable length or width

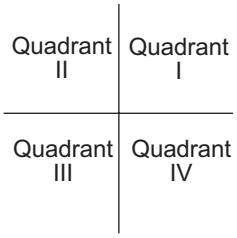
positive numbers ..... numbers greater than zero

product ..... the result of multiplying numbers together
Example: In 6 x 8 = 48,
48 is the product.

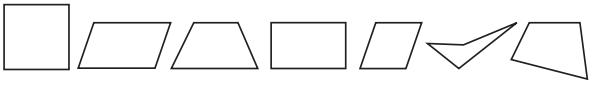
Pythagorean theorem ..... the square of the hypotenuse (c) of a right triangle is equal to the sum of the square of the legs (a and b), as shown in the equation $c^2 = a^2 + b^2$



quadrant ..... any of four regions formed by the axes in a rectangular coordinate system



quadrilateral ..... polygon with four sides
Example: square, parallelogram, trapezoid, rectangle, rhombus, concave quadrilateral, convex quadrilateral

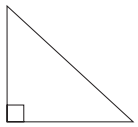


rate of change ..... how a quantity is changing over time

ratio ..... the comparison of two quantities
Example: The ratio of a and b is a:b or $\frac{a}{b}$, where $b \neq 0$.

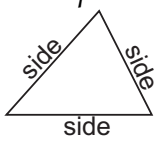
relationship ..... see *pattern*

right triangle ..... a triangle with one right angle

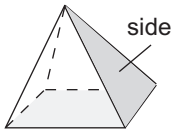


scale ..... the numeric values, set at fixed intervals, assigned to the axes of a graph

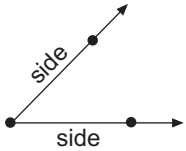
side ..... the edge of a polygon, the face of a polyhedron, or one of the rays that make up an angle
Example: A triangle has three sides.



edge of a polygon



face of a polyhedron

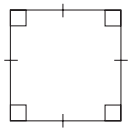


ray of an angle

slope ..... the ratio of change in the vertical axis (y-axis) to each unit change in the horizontal axis (x-axis) in the form $\frac{\text{rise}}{\text{run}}$ or $\frac{\Delta y}{\Delta x}$; the constant, m, in the linear equation for the slope-intercept form $y = mx + b$

solution ..... any value for a variable that makes an equation or inequality a true statement
Example: In $y = 8 + 9$
$y = 17$ 17 is the solution.

square ..... a rectangle with four sides the same length



square root (of a number) ..... one of two equal factors of a number
Example: 7 is the square root of 49.

substitute ..... to replace a variable with a numeral
Example: $8(a) + 3$
$8(5) + 3$

table (or chart) ..... a data display that organizes information about a topic into categories

triangle ..... a polygon with three sides; the sum of the measures of the angles is 180°

value (of a variable) ..... any of the numbers represented by the variable

variable ..... any symbol, usually a letter, which could represent a number

vertical ..... at right angles to the horizon; straight up and down

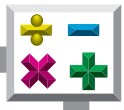
***x*-axis** ..... the horizontal number line on a rectangular coordinate system

***x*-coordinate** ..... the first number of an ordered pair

***y*-axis** ..... the vertical number line on a rectangular coordinate system

***y*-coordinate** ..... the second number of an ordered pair

***y*-intercept** ..... the value of y at the point where a line or graph intersects the y-axis; the value of x is zero (0) at this point



Unit 7: Discoveries about Linear Relationships

Introduction

The word *linear* means relating or pertaining to a line. When equations are linear equations, their graphs are lines. What are some things a linear equation could show? A linear equation could show the relationship between the number of minutes on a cell-phone plan and the total cost. A linear equation could also show the relationship between the number of math homework assignments turned in and your final grade. If the data about two variables are related in a linear way, then a linear equation can be written that describes the relationship.

Lesson One Purpose

- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand and use the real number system. (MA.A.2.4.2)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)
- Solve real-world problems involving rated measures (miles per hour, feet per second). (MA.B.2.4.2)
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)

- Describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, tables, and graphs. (MA.D.1.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)

Linear Relationships

A **pattern** or **relationship** in which there is a constant **rate of change** between two **variables** is called a *linear relationship*. The *relationship* is referred to as *linear* because the **graph** representing the **equation** is a straight **line** (↔).

There are different ways to find **solutions** to **linear equations**.

- You can make a **table** of **values** by **substituting** *values* for ***x*-coordinate** (first number of an *ordered pair*) to find values for ***y*-coordinate** (second number of an ordered pair).
- You can draw a *graph* of the *linear equation* and find **points** along that *line*.

In the next few pages we will examine *tables* that represent linear relationships.

Remember: The relationships are linear because there is a constant *rate of change*.

You will write equations to generate values for the tables by determining a value for y for a given value of x.

Practice

Each of the **tables** *on the following pages represents a* **linear relationship** *because there is a* **constant rate of change between two variables**. *For each table, you will do the following:*

- **determine** the **constant rate of change**
- **match** the **table to a descriptor**
- **write** an **equation to generate values** for the table
- **determine** a **value for** y for a given **value for** x.

Use the list below to match each **table** *with the correct* **descriptor**.

A. For each dollar spent, the amount of sales tax is 7 cents.

B. For each pound, the **equivalent** weight in ounces is 16.

C. For each yard, the *equivalent* **length** in inches is 36.

D. For each dollar spent, the total cost including sales tax is $1.06.

E. For each dollar Hetta earns, 17 cents is withheld for federal income tax.

F. For each dollar Jonathan earns, 15.3 cents is withheld for social security taxes.

G. For each hour driven, the distances traveled is 65 miles.

H. For each hour worked, $8.50 is earned in wages before taxes.

1. **Table One**

x	*y*
0	0
1	36
2	72
3	108
4	144
5	180

a. As x **increases** by 1, y *increases* by _____ .

b. Descriptor: _____

c. Equation: _____

d. When x is 12, y is _____ .

2. **Table Two**

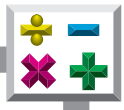
x	*y*
0	0
1	65
2	130
3	195
4	260
5	325

a. As x increases by 1, y increases by _____ .

b. Descriptor: _____

c. Equation: _____

d. When x is 8, y is _____ .



3. **Table Three**

x	*y*
0	0
1	16
2	32
3	48
4	64
5	80

a. As x increases by 1, y increases by _____ .

b. Descriptor: _____

c. Equation: _____

d. When x is 11, y is _____ .

4. **Table Four**

x	*y*
0	$0.00
1	$8.50
2	$17.00
3	$25.50
4	$34.00
5	$42.50

a. As x increases by 1, y increases by _____ .

b. Descriptor: _____

c. Equation: _____

d. When x is 40, y is _____ .

5. **Table Five**

x	*y*
$0	$0.00
$1	$0.07
$2	$0.14
$3	$0.21
$4	$0.28
$5	$0.35

a. As x increases by \$1, y increases by _____ .

b. Descriptor: _____

c. Equation: _____

d. When x is \$100, y is _____ .

6. **Table Six**

x	*y*
$0	$0.00
$100	$15.30
$200	$30.60
$300	$45.90
$400	$61.20
$500	$76.50

a. As x increases by \$100, y increases by _____ .

As x increases by 1, y increases by _____ .

b. Descriptor: _____

c. Equation: _____

d. When x is \$21,000, y is _____ .

7. **Table Seven**

x	*y*
$0	$0.00
$10	$10.60
$20	$21.20
$30	$31.80
$40	$42.40
$50	$53.00

a. As x increases by \$10, y increases by _____ .

As x increases by \$1, y increases by _____ .

b. Descriptor: _____

c. Equation: _____

d. When x is \$12, y is _____ .

8. **Table Eight**

x	*y*
$0	$0
$100	$17
$200	$34
$300	$51
$400	$68
$500	$85

a. As x increases by \$100, y increases by _____ .

As x increases by \$1, y increases by _____ .

b. Descriptor: _____

c. Equation: _____

d. When x is \$12,000, y is _____ .

Finding Consecutive and Nonconsecutive Values in Tables

Think about This!

In each of the eight tables,

- the value for y was 0 when the value for x was 0.

In each of the eight tables,

- the value for y can be found by multiplying the value for x by a constant amount.

For example, in Table One, the values for y can be found by *multiplying* the values for x by 36. When x is 6, y is 6(36) or 216.

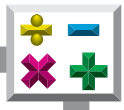
In each of the eight tables,

- **consecutive** values for y can be found by adding the constant rate of change reflected in the table.

As x increases by 1 in Table One, y increases by 36. If the y value is wanted when x is 6, 36 can be *added* to the y value when x is 5, 180 + 36 = 216.

What Was Discovered?

- When finding *consecutive values* in a table, the *additive method* may be convenient.
- When finding *nonconsecutive values* in a table, the *multiplicative method* is likely to be more efficient.



Practice

Use the list below to write the correct term for each definition on the line provided.

consecutive	**pattern (relationship)**	**value (of a variable)**
equation	**rate of change**	**variable**
equivalent (forms of a number)	**solution**	***x*-coordinate**
linear equation	**substitute**	***y*-coordinate**

_____ 1. the same number expressed in different forms

_____ 2. to replace a variable with a numeral

_____ 3. how a quantity is changing over time

_____ 4. in order

_____ 5. any of the numbers represented by the variable

_____ 6. the second number of an ordered pair

_____ 7. an algebraic equation in which the variable quantity or quantities are raised to the zero or first power and the graph is a straight line

_____ 8. any value for a variable that makes an equation or inequality a true statement

_____ 9. the first number of an ordered pair

_____ 10. a mathematical sentence in which two expressions are connected by an equality symbol

_____ 11. any symbol, usually a letter, which could represent a number

_____ 12. a predictable or prescribed sequence of numbers, objects, etc.