Lesson Two Purpose

- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand and use the real number system. (MA.A.2.4.2)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)
- Use concrete and graphic models to derive formulas for finding rate, distance, time, and angle measurements. (MA.B.1.4.2)
- Solve real-world problems involving rated measures (miles per hour, feet per second). (MA.B.2.4.2)
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)
- Using a rectangular coordinate system (graph), apply and algebraically verify properties of two-dimensional figures, including distance, midpoint, slope, parallelism, and perpendicularity. (MA.C.3.4.2)
- Describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, tables, and graphs. (MA.D.1.4.1)
- Determine the impact when changing parameters of given functions. (MA.D.1.4.2)

- Represent real-world problem situations using finite graphs. (MA.D.2.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)

Graphing Linear Relationships

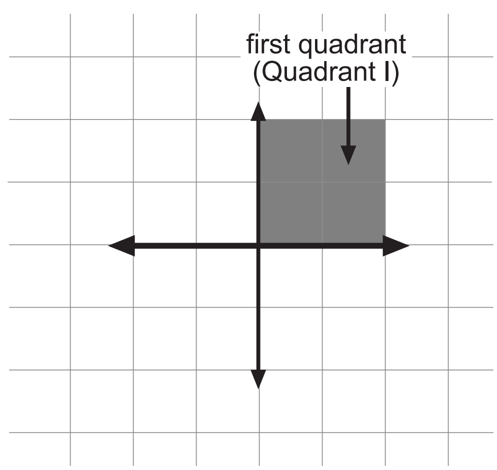
A linear relationship is a relationship in which there is a constant rate of change between two variables. A linear relationship between two variables can be represented by a straight-**line graph**.

Each of the tables in Lesson One represented linear relationships, since there was a constant rate of change between two variables. When a graph is made for each, the result will also be a straight-*line graph*.

You will make a line graph on a **coordinate grid or plane** to represent each of the relationships. First let's review how to make a graph.

Plans for Making Graphs for Tables One – Eight

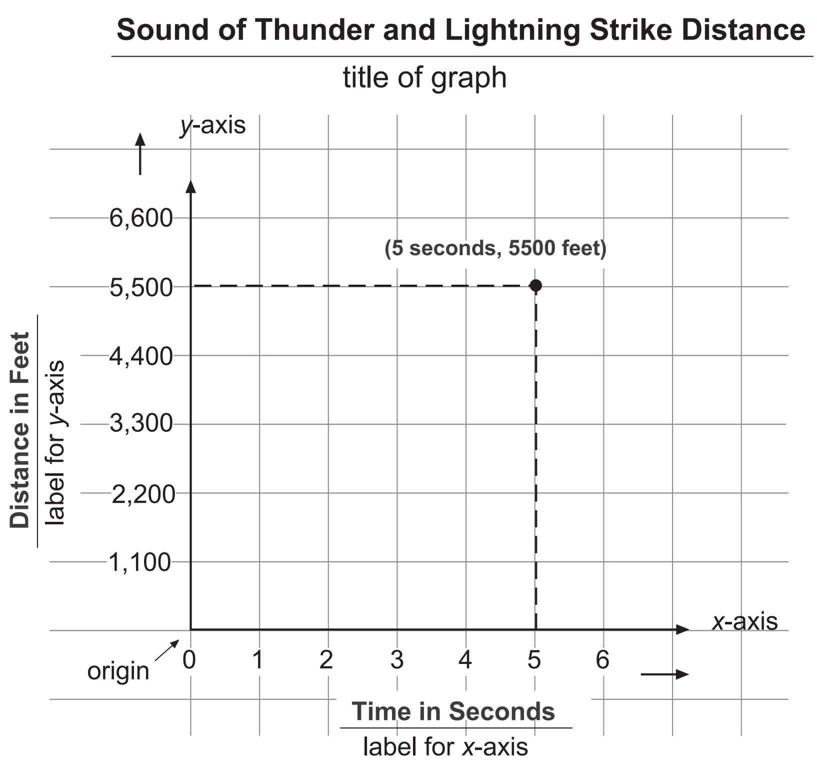
- The line graphs you will make for each of the eight tables from Lesson One contain only **positive numbers**. Thus, only the first **quadrant** or region of the graph will be needed.



coordinate graph

- Every graph will need a title. Place the title above the graph.
- On each of these eight graphs, zero (0) is used as the **minimum** value and a different **scale**, or *assigned numeric value,* has been used on each **axis**.
- Each axis on every graph will need to be appropriately **labeled**.
- The **ordered pairs**, or *points* for the data on each of the eight graphs, start at the **coordinates** of the **origin** (0, 0). The *origin* is the **intersection** where the ***x*-axis** and the ***y*-axis** meet. First, locate the number on the *x-axis*. From that point on the *x*-axis, move straight up and **parallel (||)** to the *y-axis*. Move to the point aligned with the correct number on the *y*-axis and draw a point.

For example, lets suppose you were graphing the time between seeing lightning and hearing the thunder. See the graph below. The first set of *ordered pairs* (5, 5500) has been located for you. The 5 is the first number of the ordered pair, or the *x*-coordinate on the *x*-axis (⟷). The 5,500 is the second number of the ordered pair, or the *y*-coordinate.



Think about This!

Since we know each table in Lesson One represents a linear relationship, it is not necessary to plot each of the six points reflected in each table.

- Two points determine a line.
- A third point provides a check for accuracy in the other points.

We know errors can be made when we determine values in a table or when we plot points. Plot at least 3 points on each graph that represent **data** in the tables. By visual inspection, verify that the points you did not plot lie on your straight-line graph.

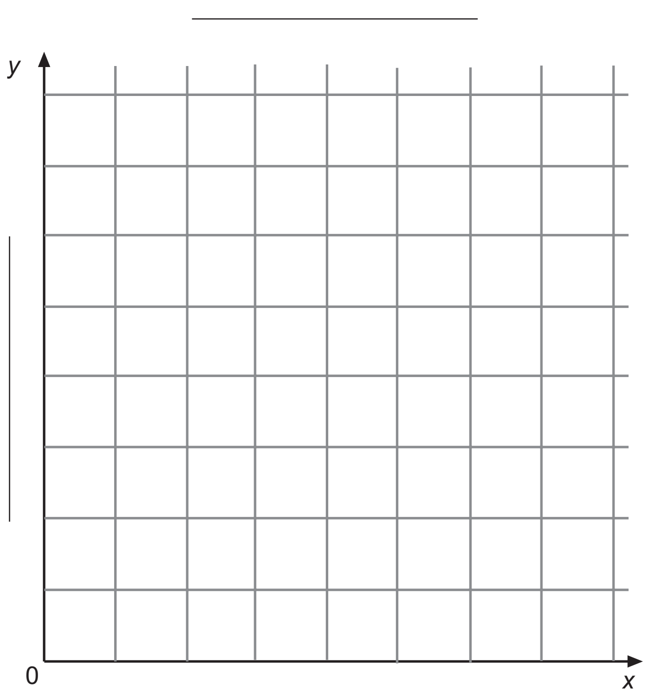
Practice

Make a graph for each of the eight tables from **Lesson One**. *Be sure to do the following:*

- **Title** *each graph.*
- **Label** *the axes.*
- *Use an appropriate* **scale**.
- **Connect** *the* **points** *in each graph to illustrate the* **linear relationship**.

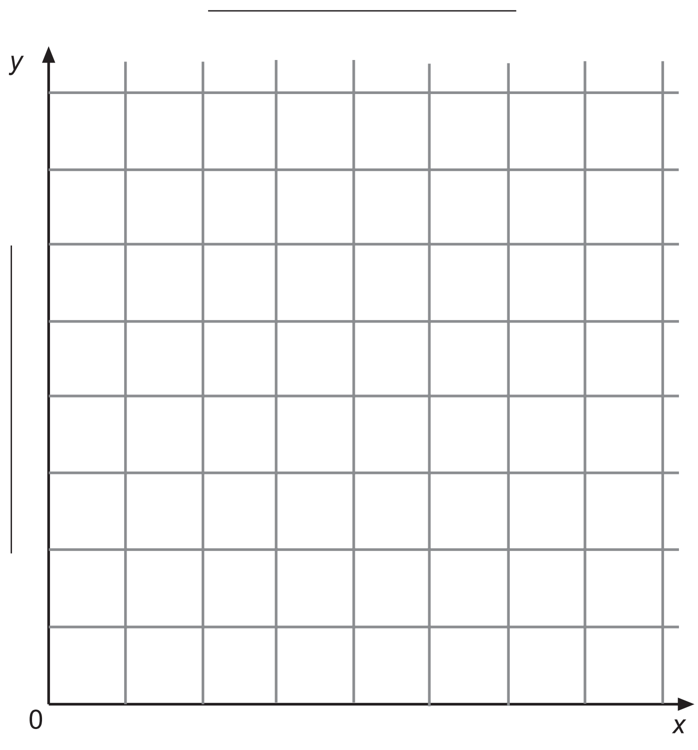
1. **Table One**

x	*y*
0	0
1	36
2	72
3	108
4	144
5	180



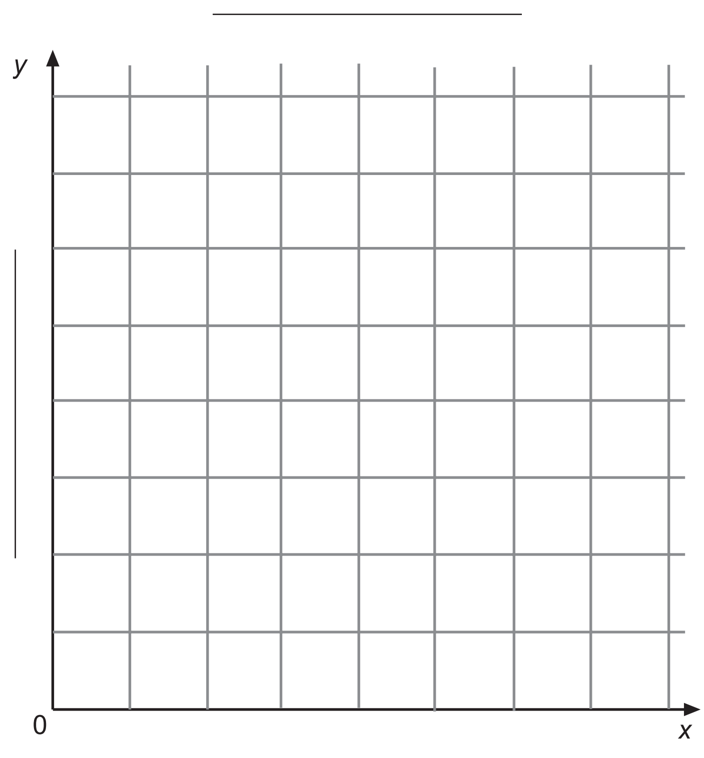
2. **Table Two**

x	*y*
0	0
1	65
2	130
3	195
4	260
5	325



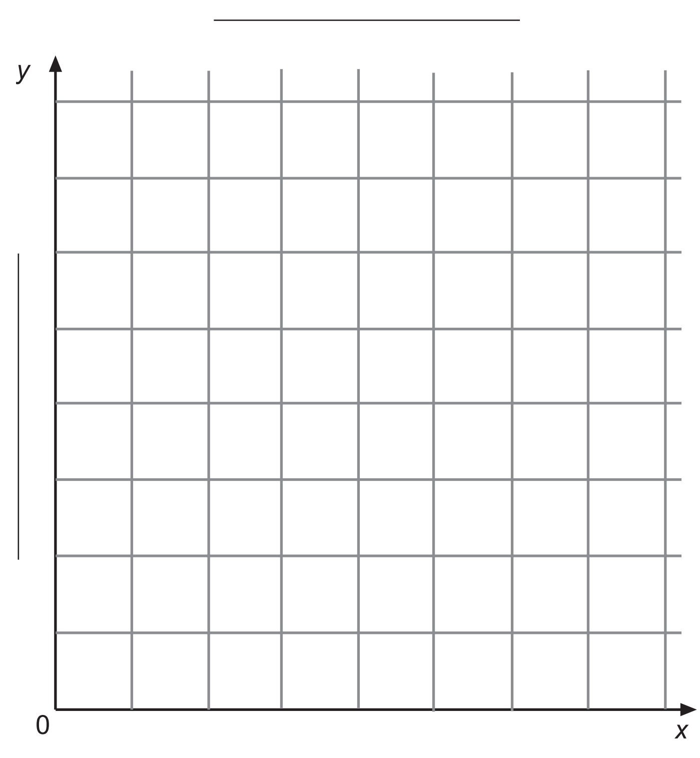
3. **Table Three**

x	*y*
0	0
1	16
2	32
3	48
4	64
5	80



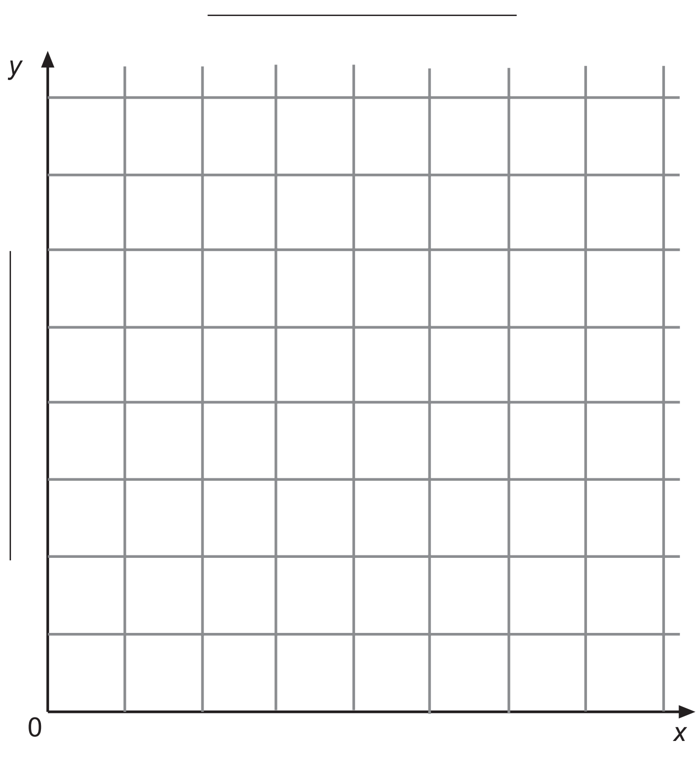
4. **Table Four**

x	*y*
0	$0.00
1	$8.50
2	$17.00
3	$25.50
4	$34.00
5	$42.50



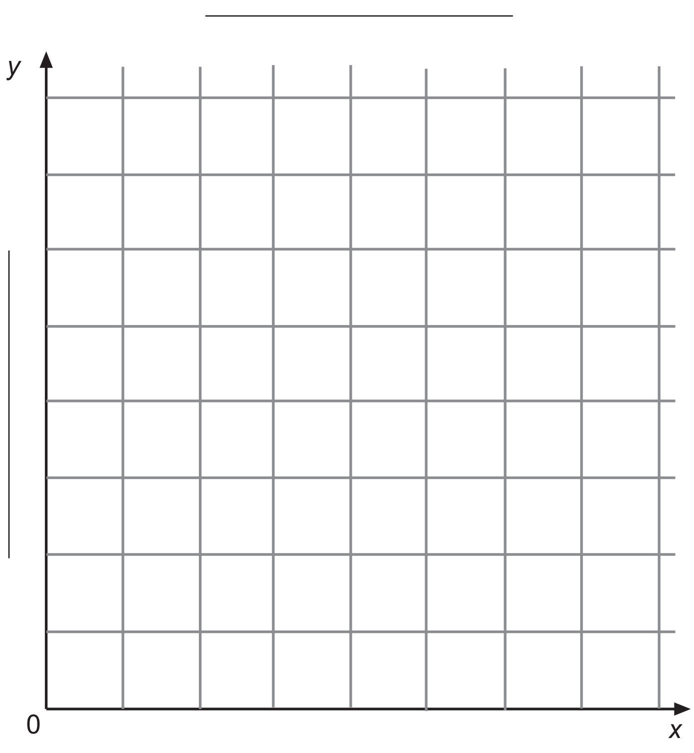
5. **Table Five**

x	*y*
$0	$0.00
$1	$0.07
$2	$0.14
$3	$0.21
$4	$0.28
$5	$0.35



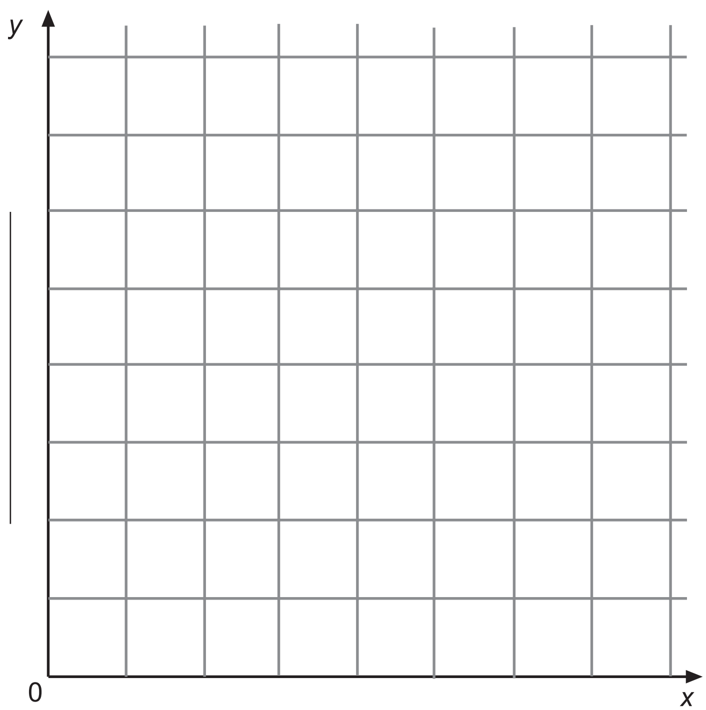
6. **Table Six**

x	*y*
$0	$0.00
$100	$15.30
$200	$30.60
$300	$45.90
$400	$61.20
$500	$76.50



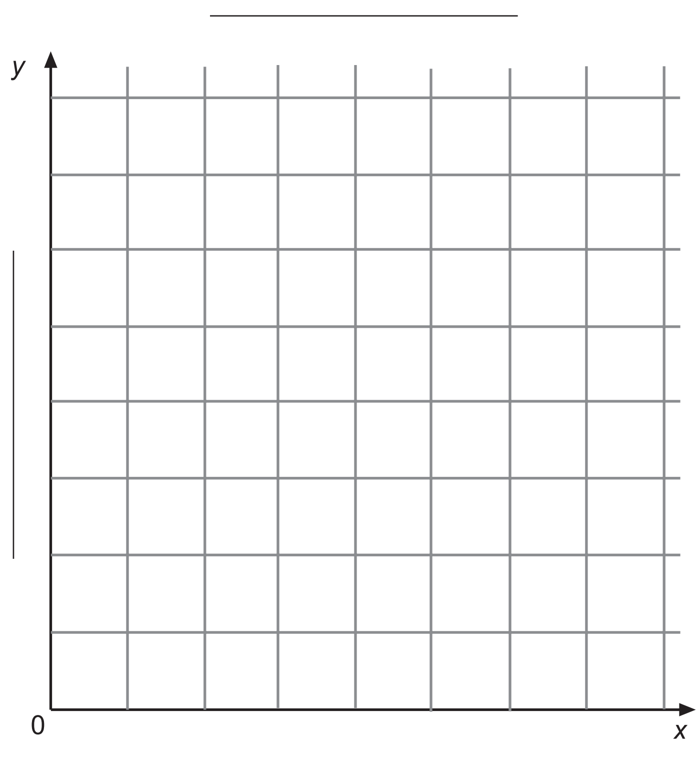
7. **Table Seven**

x	*y*
$0	$0.00
$10	$10.60
$20	$21.20
$30	$31.80
$40	$42.40
$50	$53.00



8. **Table Eight**

x	*y*
$0	$0
$100	$17
$200	$34
$300	$51
$400	$68
$500	$85



Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. a drawing used to represent data

______ 2. the horizontal and vertical number lines used in a coordinate plane system

______ 3. the horizontal number line on a rectangular system

______ 4. the vertical number line on a rectangular system

______ 5. a graph that displays data using connected line segments

______ 6. information in the form of numbers gathered for statistical purposes

______ 7. numbers that correspond to points on a coordinate plane in the form (x, y)

______ 8. the location of a single point on a rectangular coordinate system where the first and second values represent the position relative to the x-axis and y-axis, respectively

______ 9. the point of intersection of the x- and y-axes in a rectangular coordinate system, where the x-coordinate and y-coordinate are both zero (0)

______ 10. the point at which two lines or curves meet

A. axes (of a graph)

B. coordinates

C. data

D. graph

E. intersection

F. line graph

G. origin

H. ordered pair

I. x-axis

J. y-axis

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y-Intercept Form

As discussed earlier, a linear relationship is a relationship in which there is a constant rate of change between two variables. This relationship can be represented by a straight-line graph. A linear relationship can also be represented by an equation of the form: $y = mx + b$.

The rate of change is the **coefficient**, m, of x. The b in the equation is a constant. It indicates where the graph crosses the y-axis $(0, b)$. This point is called the ***y*-intercept**.

Think about This!

The equations for the eight tables in lesson one were of the form

$$y = mx + b.$$

We also understand that in Table One

$$y = 36x$$

is equivalent to

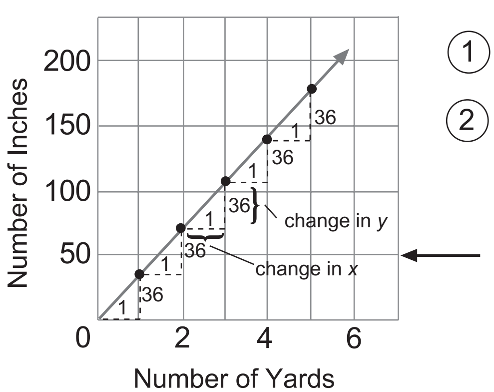
$$y = 36x + 0.$$

Table One

x	*y*
0	0
1	36
2	72
3	108
4	144
5	180

We can verify the rate of change reflected in the table is 36 and that for an increase of 1 in the value for x, there is an increase of 36 in the value for y on the graph. We can also verify that the line crosses the y-axis at (0, 0).

$y = 36x$

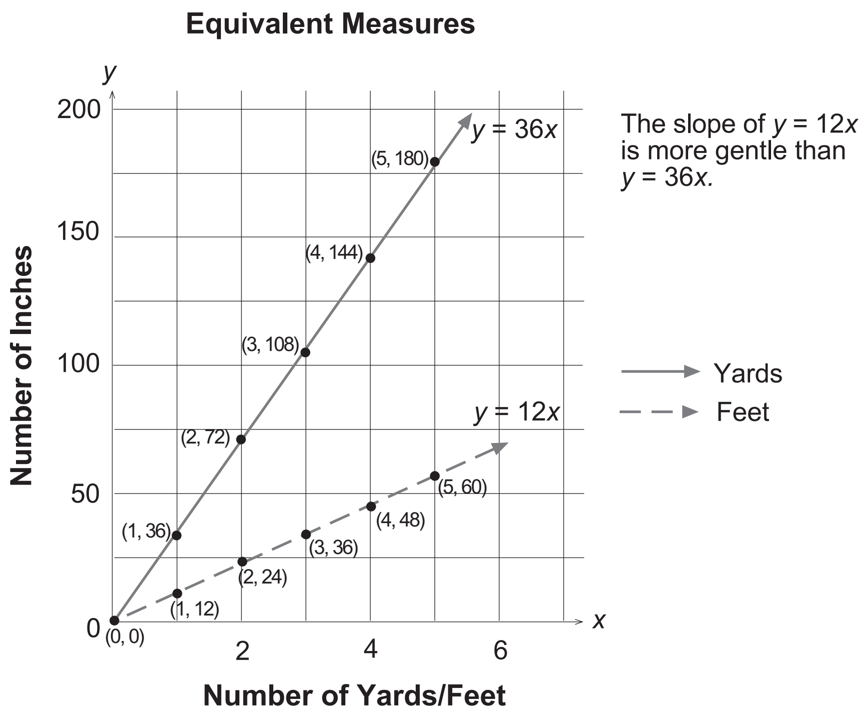


① lot *y*-intercept at (0, 0).

② From *y*-intercept, move horizontally 1 unit to increase *x* by 1 and vertically to increase *y* by 36.

For each increase of 1 in number of yards, there is a corresponding increase of 36 in the number of inches.

If we were to use the same *coordinate grid* to make a line graph showing equivalent measures for feet and inches, the equation would be represented as $y = 12x$ or $y = 12x + 0$ when x represents the number of feet and y represents the number of inches. For each increase of 1 in the value for x, there would be an increase of 12 in the value for y. The **slope** of this line would be more gentle than the *slope* of the line $y = 36x$. The y-*intercept* would be (0, 0). These statements are illustrated on the graph below.



Practice

Complete the following.

Brianna obtained prices from two companies for the printing of wedding invitations.

Remember: We are using this equation of the form

$y = mx + b$

1. Company A charges $3.00 per invitation.

   a. Write an equation that could be used to determine cost of any number of invitations where y represents the total cost and x represents the number of invitations.

   ____

   ____

   b. Make a table for cost for 0-500 invitations counting by 100.

Company A—Cost of Wedding Invitations

x (Number of invitations)	y (Cost at Company A)
0	$
100	$
200	$
300	$
400	$
500	$

2. Company B charges a set-up fee of $50 and $2.50 per invitation.

a. Write an equation that could be used to determine cost of any number of invitations when y represents the costs and x represents the number of invitations.

____

____

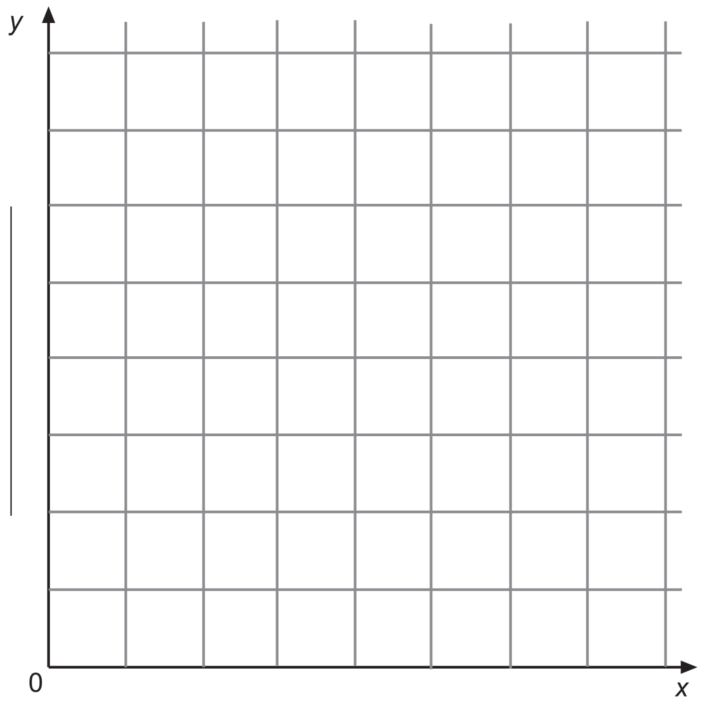
b. Make a table for cost for 0-500 invitations counting by 100.

Company B—Cost of Wedding Invitations

x (Number of invitations)	*y* (Cost at Company B)
0	$
100	$
200	$
300	$
400	$
500	$

3. Graph the data from each company on the same coordinate grid.

Cost of Wedding Invitations—Company A and B



4. Refer to your equations in numbers 1 and 2 for Company A and Company B.

The coefficient of x in the equation for Company A is

____________ , and ____________ for Company B. For each

100 invitations from Company A, the rate of change is

____________ . For each 1 invitation, the rate of change

is ____________ . For each 100 invitations from

Company B, the rate of change is ____________ . For each 1

invitation, the rate of change is ____________ .

5. The line graph for Company A has a greater slope than the one for Company B. Explain why. ____________________________________

____

____

____

6. For which company is the y-intercept (0, 0)? ____________________

How does this show up in the equation? ______________________

____

____

The table? ____

____

The graph? ____

____

7. For which company is the y-intercept (0, 50)? ____________________

How does this show up in the equation? ________________________

____

____

The table? ____

____

The graph? ____

____

8. The two lines **intersect** (meet) at (100, 300). Explain why this is true.

____

____

____

____

____

Practice

Complete the following.

The number of times a cricket chirps in a minute depends on the temperature. The **formula** is

$$y = 4x - 160.$$

when y represents the number of chirps and x represents the temperature in degrees Fahrenheit (F).

Remember: We are using the equation of the form

$$y = mx + b.$$

1. If a table of values is made, the values for y should increase

   ____________ for each increase of 1 in the values of x.

   Complete the following table to verify your answer.

Temperature and Number of Times a Cricket Chirps

x (Temperature in Degrees Fahrenheit)	y (Number of Chirps)
0° F	
20° F	
40° F	
60° F	
80° F	
100° F	

2. In the real world, a cricket chirps or does *not* chirp. There is no such thing as a **negative number** of chirps. The y-intercept for this equation has little meaning when applied to the context of the problem, but it does have meaning when we compare the equation to the table of values and to the graph.

   If we graphed the relationship of chirps to temperature, the y-intercept would be ____________ . For each increase of 20 degrees F in temperature (above 40), the number of chirps increases by ____________ . For each increase of 1 degree F temperature (above 40), the number of chirps increases by ____________ .

3. How does the y-intercept show up in the equation? ____________

   ____

   ____

   The table? ____

   ____

   How would it show up in the graph? __________________________

   ____

4. How does the constant rate of change show up in the equation?

____

____

The table? ______________________________________

____

How would it show up in the graph? ______________

____

5. When you graph this relationship, follow these steps:

- Title your graph.
- Label your axes.
- Using the values from the table as a guide, the *scale* for your x-axis will likely number from 0-100 counting by 20s. The scale for your y-axis will likely number from (-160) to 240 counting by 40s.
- Plot the point for the y-intercept, (0, ______).
- From that point, increase the x-value by 20 moving **horizontally (⟷)** to the right, and then increase the y-value by 40 moving **vertically (↕)** upward. Plot your second point at this location.
- Repeat the last step by movement from the 2nd point plotted. Your points should lie in a straight line.
- You could continue doing this or you could confidently connect the three points with a line. The values for each and every point through which the line passes would make the equation *true*.

Graphs and Linear Relationships

You have practiced more than one way to make a graph for a linear relationship.

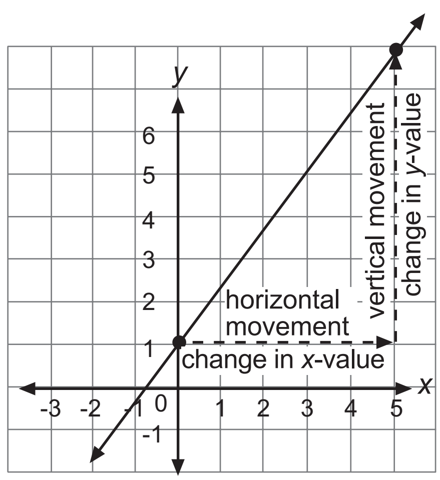
- One way is to make a table of at least three values for x and the corresponding values for y and to plot points represented by those ordered pairs.
- Another way is to determine the y-intercept by determining the value for y when the value for x is 0. You can now plot the point where the graph will cross the y-axis.

When the equation is in the form of $y = mx + b$, you know the *coefficient* of the x term represents the *constant rate of change*.

- From the y-intercept,

  horizontal movement can represent the change in the x-value

  followed by vertical movement to represent the corresponding change in the y-value.



slope of a line

The steepness of a line is called its *slope*. The *vertical* (↕) change is called the *change in y* and the *horizontal* (⟷) change is called the *change in x*.

Examine the following equation.

$$y = 2x + 6$$

The coefficient of x is 2, which is equivalent to $\frac{2}{1}$. The *first number* in this special **ratio** (2) represents the *vertical change* and the *second number* (1) represents the *horizontal change*. This is often referred to as $\frac{\text{rise}}{\text{run}}$.

$$y = 2x + 6$$

coefficent of x is equivalent to $\frac{2}{1} = \frac{\text{represents the vertical } (\updownarrow) \text{ change}}{\text{represents the horizontal } (\leftrightarrow) \text{ change}} = \frac{\text{rise}}{\text{run}}$

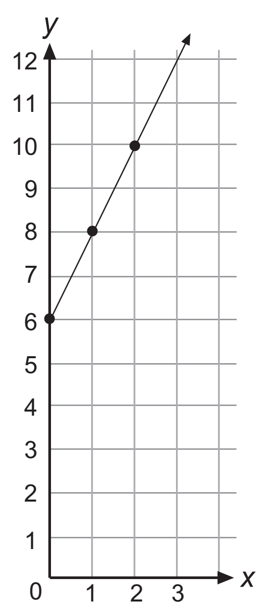
Consider the table to the right.

Table of Values

$y = 2x + 6$	
x	**y**
0	6
1	8
2	10

For the equation $y = 2x + 6$, you might make a graph either of the following ways.

Graph of $y = 2x + 6$ *or* **Graph of $y = 2x + 6$**

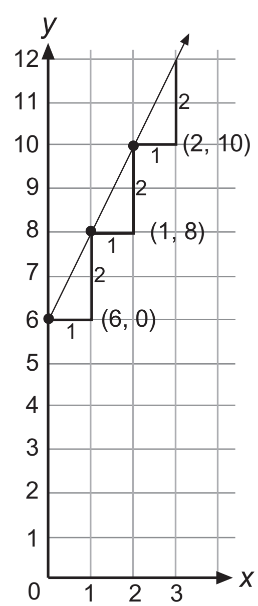


Graph 1

y-intercept:
$y = mx + b$
$(0, b)$
$(0, 6)$

slope *or* rate of change
$y = mx + b$
$y = 2x + 6$

For each increase of 1 in x there will be an increase of 2 in y.



Graph 2

y-intercept:
$y = mx + b$
$(0, b)$
$(0, 6)$

slope *or* rate of change
$y = mx + b$
$y = 2x + 6$

For each increase of 1 in x there will be an increase of 2 in y.

Practice

Complete the following.

For each of the following equations, do the following.

- Determine the **slope**.
- Determine the y-intercept.
- Use the slope and y-intercept to make a graph for each equation.

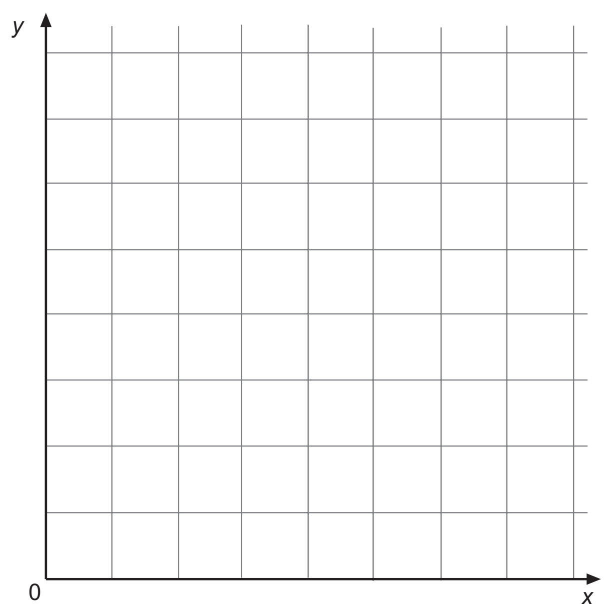
1. $y = 3x + 7$

   For each increase of 1 in the value of x (horizontal change ⟷), there will be an increase of ____________ for the corresponding value of y (vertical change ↕).

   The slope is represented by the coefficient of x in the equation which is 3. The special *ratio* is therefore $\frac{3}{1}$. For each horizontal increase of 1, there will be a vertical increase of 3.

   The y-intercept is (0, ______ .)

Graph of $y = 3x + 7$



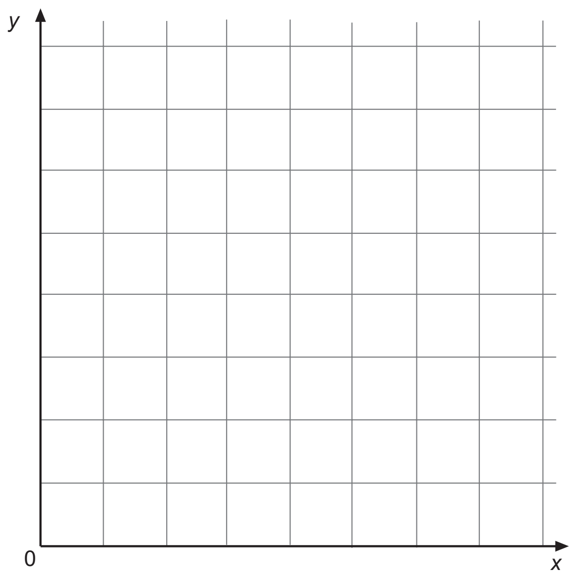
2. $y = 2x - 5$

For each increase of 1 in the value of x, there will be an increase of

____________ for the corresponding value of y.

The y-intercept is (0, ______ .)

Graph of $y = 2x - 5$



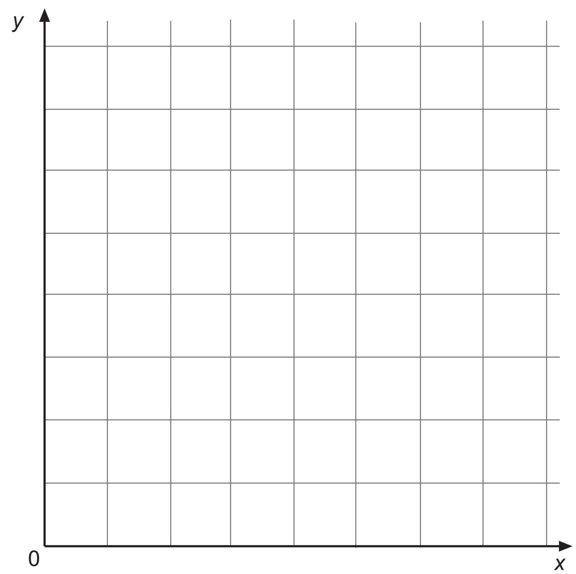
3. $y = 4x$

For each increase of 1 in the value of x, there will be an increase of

_____________ for the corresponding value of y.

The y-intercept is (0, _______ .)

Graph of $y = 4x$



Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. to meet or cross at one point

______ 2. a way of expressing a relationship using variables or symbols that represent numbers

______ 3. the value of y at the point where a line or graph intersects the y-axis; the value of x is zero (0) at this point

______ 4. at right angles to the horizon; straight up and down

______ 5. the ratio of change in the vertical axis (y-axis) to each unit change in the horizontal axis (x-axis) in the form $\frac{\text{rise}}{\text{run}}$ or $\frac{\Delta y}{\Delta x}$

______ 6. numbers less than zero

______ 7. the comparison of two quantities

______ 8. parallel to or in the same plane of the horizon

A. formula

B. horizontal

C. intersect

D. negative numbers

E. ratio

F. slope

G. vertical

H. y-intercept