Lesson Four Purpose

- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand and use the real number system. (MA.A.2.4.2)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)
- Using a rectangular coordinate system (graph), apply and algebraically verify properties of two-dimensional figures, including distance, midpoint, slope, parallelism, and perpendicularity. (MA.C.3.4.2)
- Describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, tables, and graphs. (MA.D.1.4.1)
- Determine the impact when changing parameters of given functions. (MA.D.1.4.2)
- Represent real-world problem situations using finite graphs. (MA.D.2.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)

Slope of Parallel and Perpendicular Lines

In this section we will explore slopes of **parallel lines** and **perpendicular lines**. Lines with positive slopes and negative slopes will also be explored.

Finding the Slope of a Line

One way to determine the slope of a line is to do the following:

- find the vertical (↕) change by finding the **difference** in *y*-values for two designated points on the line
- find the horizontal (↔) change by finding the *difference* in corresponding *x*-values for those two designated points.

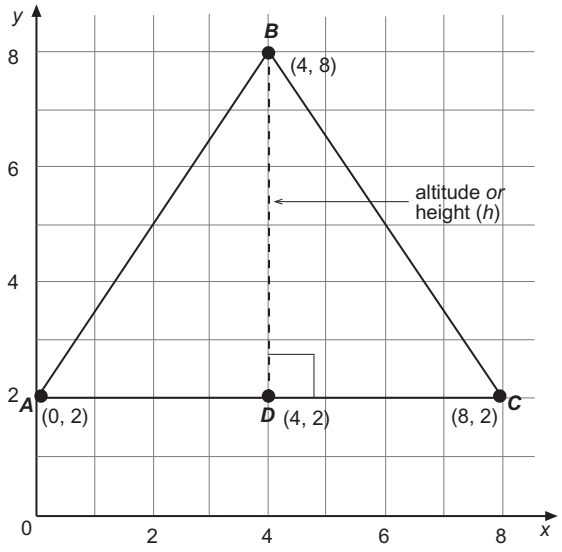
The *formula* is often presented as

$$\text{slope } (m) = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Figure 1 below allows us to examine the slopes of the three **sides** as well as the **altitude**, or **height (*h*)**, of an **isosceles triangle** drawn on a coordinate grid.

Remember: An *isosceles triangle* is a **triangle** with two **congruent (≅)** *sides* and two *congruent* **angles**.

Figure 1



The slope of **line segment (—)** AB ($\overline{AB}$) can be found using the formula

$$\text{slope } (m) = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Remember: A *line segment* is a part of a line. It can be referred to either as a line segment or a *segment*. A line segment has two **endpoints** and includes all points between those two *endpoints*. To name a line segment, use its endpoints.

- Use the symbol (—) over the two endpoints—$\overline{AB}$.
- You can write—line segment AB.
- Or you can write—segment AB.

All three mean the distance between point A and point B.

In the formula

$$\text{slope } (m) = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1},$$

the little numbers under the y's and the x's above, (x_1, y_1) and (x_2, y_2), are called *subscripts.* The subscripts are a way of letting us know that we are talking about *point 1* and *point 2* and that they are *different* points. The subscripts mean that these are two distinct points of the form (x, y).

Note: The subscripts are *not* exponents.

The coordinates for point A will represent the first ordered pair and the coordinates for point B will represent the second ordered pair.

point A $(0, 2)$ point B $(4, 8)$

$$\text{slope } (m) = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 2}{4 - 0} = \frac{6}{4} \text{ or } \frac{3}{2}$$

The slope for segment *BC* is as follows.

point *B* (4, 8) point *C* (8, 2)

$$\text{slope } (m) = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 8}{8 - 4} = \frac{-6}{4} \text{ or } \frac{-3}{2}$$

The slope for segment *CA* is as follows.

point *C* (8, 2) point *A* (0, 2)

$$\text{slope } (m) = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{0 - 8} = \frac{-0}{-8} \text{ or } 0$$

The slope for segment *BD* is as follows.

point *B* (4, 8) point *D* (4, 2)

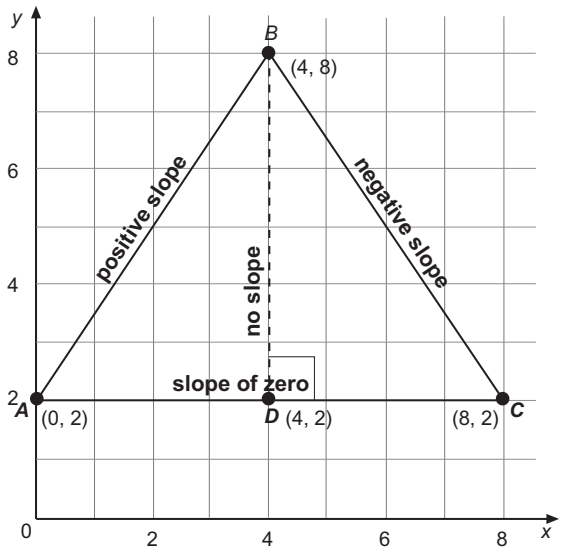
$$\text{slope } (m) = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 8}{4 - 4} = \frac{\cancel{-6}}{\cancel{0}}$$

Since $\frac{-6}{0}$, or division by zero, is *undefined* in mathematics, segment *BD* is said to have *no* slope.

Findings about the Slopes

Look at Figure 1 from page 450, redrawn below. The slope of each segment has been marked as follows:

Figure 1



- Segment *CA* has a slope of zero.

  All *horizontal* lines have a slope of *zero*.

- Segment *BD* has no slope.

  A *vertical* line has *no* slope.

- Segment *AB* has a slope of $\frac{3}{2}$, which is positive.

  All lines that *rise* from *left to right* have *positive* slopes.

- Segment *BC* has a slope of $\frac{-3}{2}$, which is negative.

  All lines that *rise* from *right to left* have *negative* slopes.

  or

  All lines that *fall* from *left* to *right* have negative slopes.

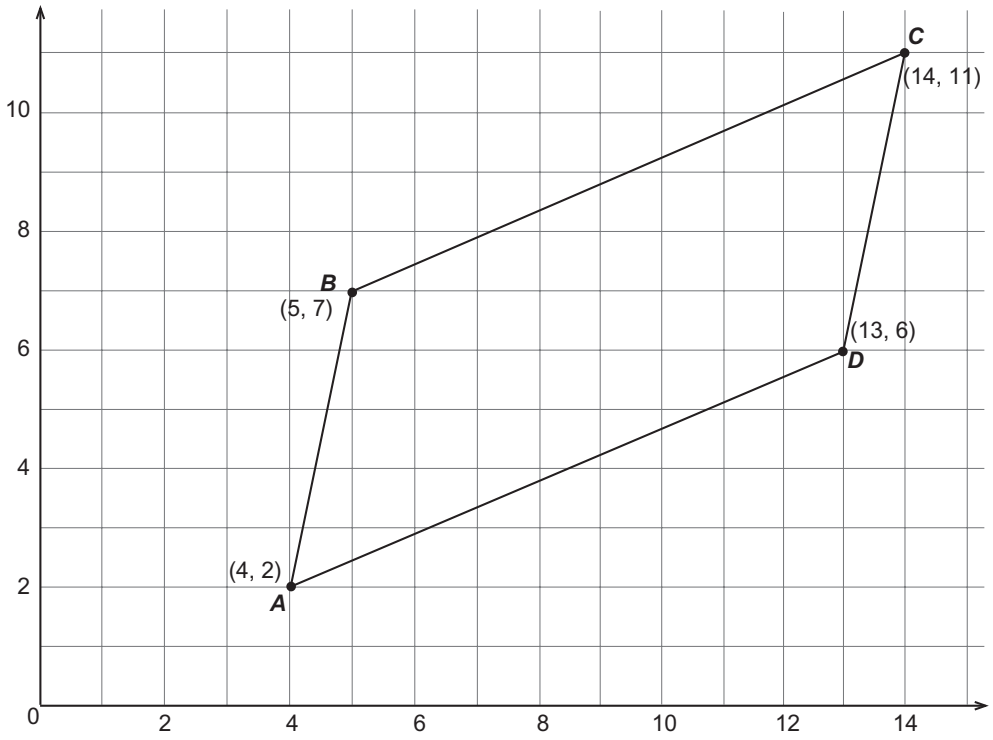
Practice

Complete the following.

Figure 2 allows you to examine **slopes** of the four **sides** of a **parallelogram** drawn on a coordinate grid.

Remember: A *parallelogram* is a **quadrilateral** with two pairs of **parallel** *sides*.

Figure 2



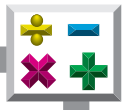
1. Find the slope of each of the four sides of the parallelograms.

   a. The slope of segment *AB* is _____ .

   b. The slope of segment *BC* is _____ .

   c. The slope of segment *CD* is _____ .

   d. The slope of segment *DA* is _____ .



2. All four sides of parallelogram *ABCD* rise from left to right and all four have _____ (positive/negative) slopes.

3. Sides *AB* and *DC* are opposite each other. We know that **opposite sides** of a parallelogram are parallel. The slope of each side is _____ . Slopes of *parallel lines* _____ (are, are not) equal.

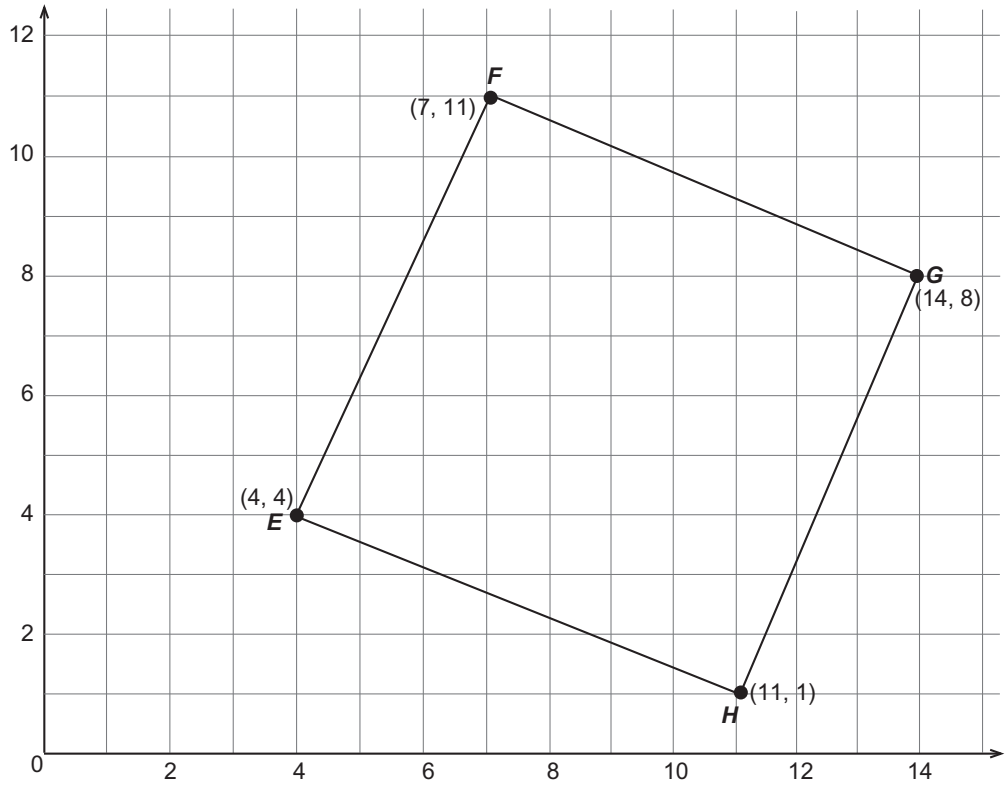
4. Sides *BC* and *DA* are opposite each other. We know that *opposite sides* of a parallelogram are parallel. The slope of each side is _____ . Slopes of parallel lines _____ (are, are not) equal.

5. Sides *AB* and *BC* intersect each other. Their slopes _____ (are, are not) the same. Slopes of intersecting lines _____ (are, are not) equal.

Complete the following.

Figure 3 allows you to examine **slopes** of the four **sides** of **square** *EFGH* drawn on a coordinate grid.

Figure 3



6. Find the slope of each of the four sides of square *EFGH*.

   a. The slope of segment *EF* is _____ .

   b. The slope of segment *FG* is _____ .

   c. The slope of segment *GH* is _____ .

   d. The slope of segment *HE* is _____ .

7. Sides *EF* and *GH* are opposite each other. We know that opposite sides of a square are parallel. The slope of each side is _____ . Slopes of parallel lines _____ (are, are not) equal.

8. Sides *FG* and *HE* are opposite each other. We know that opposite sides of a square are parallel. The slope of each side is _____ . Slopes of parallel lines _____ (are, are not) equal.

9. Sides *EF* and *FG* intersect each other. Their slopes _____ (are, are not) equal.

10. Sides *EF* and *FG* intersect each other at right angles since the figure is a square. You know that **adjacent sides** of a square are perpendicular (⊥) to each other. The slope of segment *EF* is _____ and the slope of segment *FG* is _____ . The **product** of their slopes _____ (is, is not) -1.

11. Sides *EF* and *HG* rise from left to right. Their slopes _____ (are, are not) positive.

12. Sides *GF* and *HE* rise from right to left. Their slopes _____ (are, are not) positive.

Findings about Lines and Their Slopes

The findings made in this lesson about lines and their slopes are important. They will be applied often in your later study of geometry. Review the following statements; you will need the information for future reference.

- ***Parallel* lines have the *same* slope.**
- ***Vertical* lines have *no* slope and are known to be parallel.**
- **The slopes of two lines that are perpendicular to each other will have a product of -1.**
- **Two lines in the same plane having different slopes will intersect.**
- **Lines *rising* from *left* to *right* will have positive slopes.**
- **Lines *rising* from *right* to *left* will have negative slopes**

  or

  Lines *falling* from *left* to *right* will have negative slopes.

Practice

Use the list below to write the correct term for each definition on the line provided.

adjacent sides	**line segment (—)**	**parallelogram**
endpoint	**opposite sides**	**perpendicular lines**
isosceles triangle	**parallel lines**	**side**

_____ 1. two lines in the same plane that are a constant distance apart; lines with equal slopes

_____ 2. a portion of a line that consists of two defined endpoints and all the points in between

_____ 3. either of two points marking the end of a line segment

_____ 4. two lines that intersect to form right angles

_____ 5. sides that are next to each other and share a common vertex

_____ 6. sides that are directly across from each other

_____ 7. the edge of a polygon

_____ 8. a triangle with two congruent sides and two congruent angles

_____ 9. a quadrilateral with two pairs of parallel sides