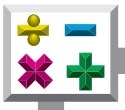


Lesson Five Purpose

- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand and use the real number system. (MA.A.2.4.2)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)
- Use concrete and graphic models to derive formulas for finding rate, distance, time, and angle measurements. (MA.B.1.4.2)
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)
- Using a rectangular coordinate system (graph), apply and algebraically verify properties of two-dimensional figures, including distance, midpoint, slope, parallelism, and perpendicularity. (MA.C.3.4.2)
- Describe, analyze, and generalize relationships, patterns, and functions using words, symbols, variables, tables, and graphs. (MA.D.1.4.1)
- Determine the impact when changing parameters of given functions. (MA.D.1.4.2)
- Represent real-world problem situations using finite graphs. (MA.D.2.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)

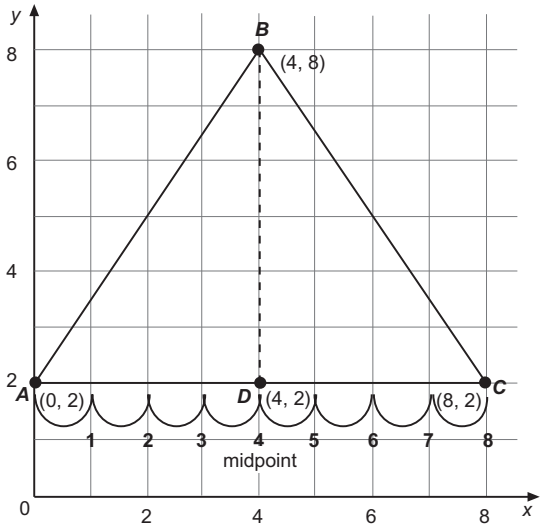
Midpoints

You will use Figure 1 on page 450, Figure 2 on page 454, and Figure 3 on page 456 as you explore finding the **midpoint** of a line segment and finding the distance between two points on a coordinate grid. These procedures will be applied many times in your future study of mathematics.

- In triangle (△) *ABC* of Figure 1 (see below), the *midpoint* of segment *AC* could easily be found since it is a horizontal line segment.

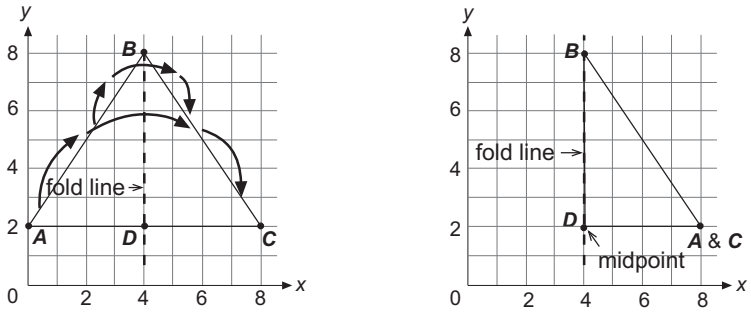
  You could find it by counting.

Figure 1



finding midpoint by counting

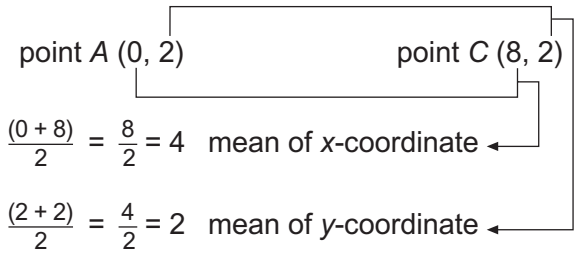
You could also fold the segment so that the two endpoints were exactly on top of each other and the fold would be at the midpoint.



finding midpoint by folding

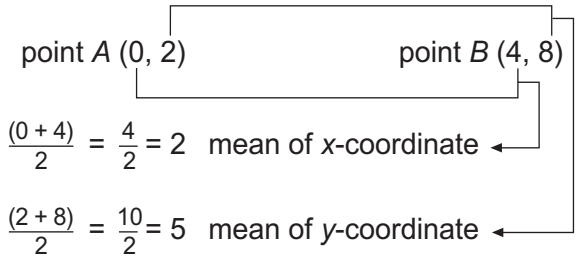
- Another way is to find the **mean** of the x coordinates for points A and C and the *mean* of their y-coordinates.

  Finding the midpoint of segment AC.



The coordinates for the midpoint of segment AC will be (4, 2). This can be verified by visual inspection.

Find the midpoint of segment AB.

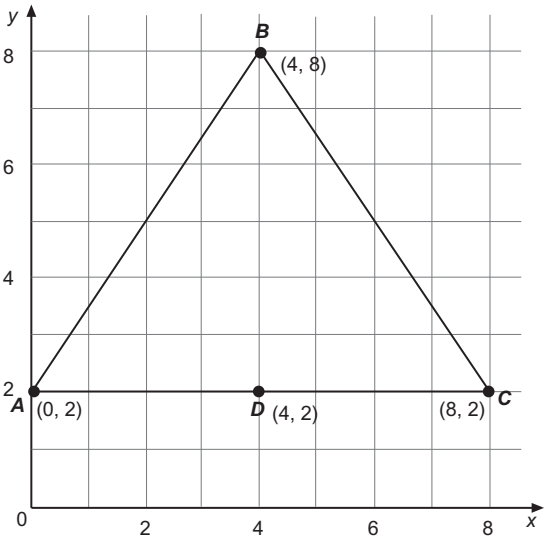


The coordinates for the midpoint of segment AB will be (2, 5). This can be verified by visual inspection.

Practice

Complete the following.

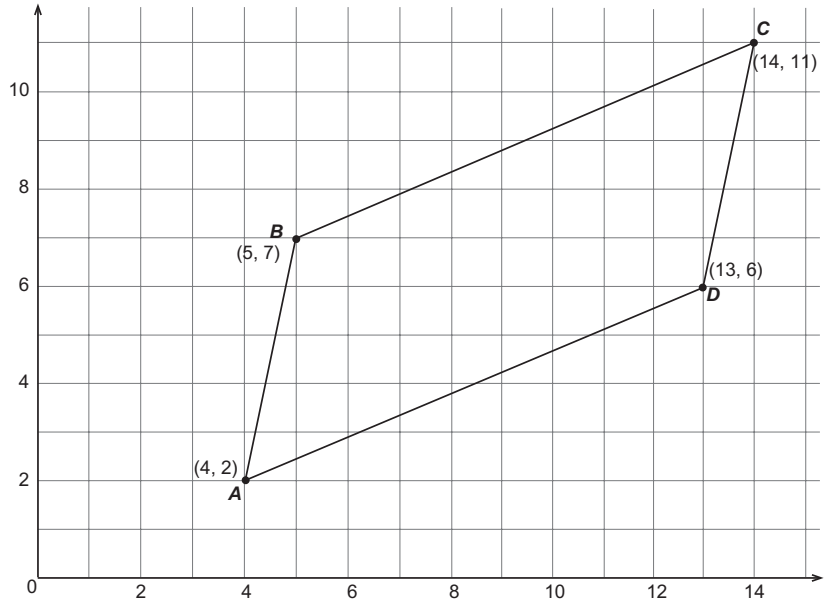
Figure 1



1. Find the midpoint of segment *BC*.

   _____

Figure 2



2. Find the midpoint of each side of the parallelogram.

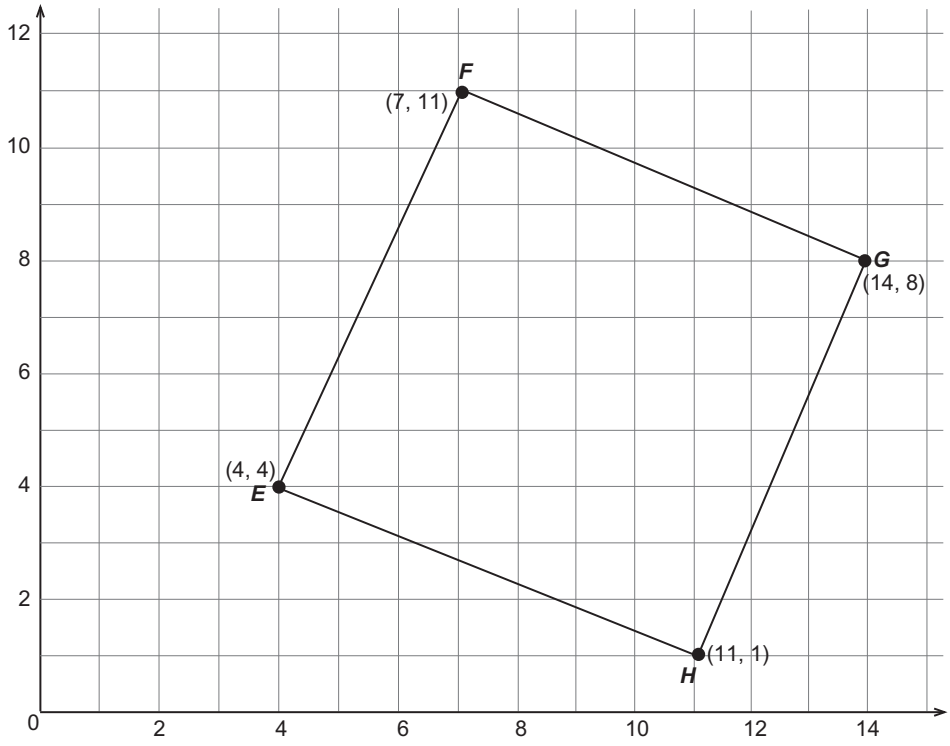
   a. The midpoint of segment *AB* is _____ .

   b. The midpoint of segment *BC* is _____ .

   c. The midpoint of segment *CD* is _____ .

   d. The midpoint of segment *DA* is _____ .

Figure 3



3. Find the midpoint of each side of the parallelogram.

   a. The midpoint of segment *EF* is _____ .

   b. The midpoint of segment *FG* is _____ .

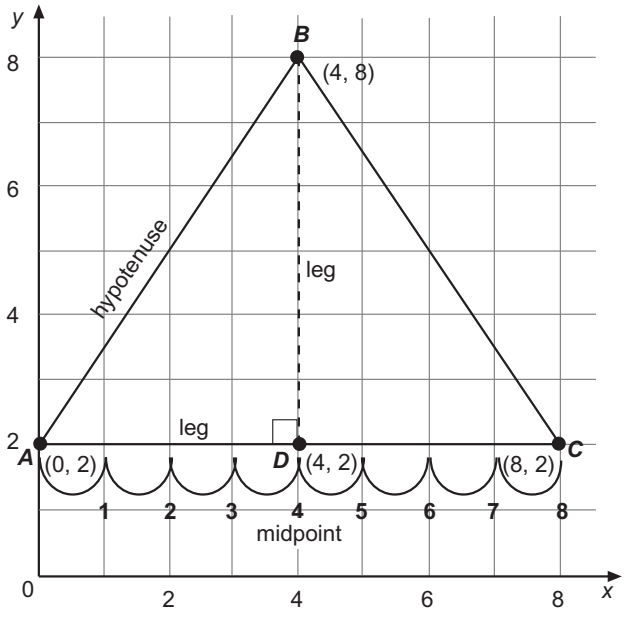
   c. The midpoint of segment *GH* is _____ .

   d. The midpoint of segment *HE* is _____ .

Finding the Distance between Two Points

Consider again, triangle *ABC* below. The distance from point *A* to point *C* could easily be found by counting since this segment is horizontal.

Figure 1



finding midpoint by counting

The distance between points *A* and *B* is a bit more challenging to determine, especially if we are not using a straightedge with appropriate unit markings.

If you let segment *AB* represent the **hypotenuse** of a **right triangle**, the two **legs** could be sketched. One would extend from (4, 2) to (4, 8) and it would have length of 6 units. The other would extend from (0, 2) to (4, 2) and it would have length of 4 units.

We know $a^2 + b^2 = c^2$.

Therefore
$$6^2 + 4^2 = c^2$$
$$36 + 16 = c^2$$
$$52 = c^2$$

If $c^2 = 52$, the c = the **square root** of 52.
$$c = \sqrt{52}$$

There is a formula for determining the distance (d) between two points on a coordinate grid and it is as follows:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

As you use the formula, consider its relationship to the use of the **Pythagorean theorem** ($c^2 = a^2 + b^2$).

To use this formula to find the length of segment AB in triangle ABC, you would do the following:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(4 - 0)^2 + (8 - 2)^2}$$

$$d = \sqrt{16 + 36}$$

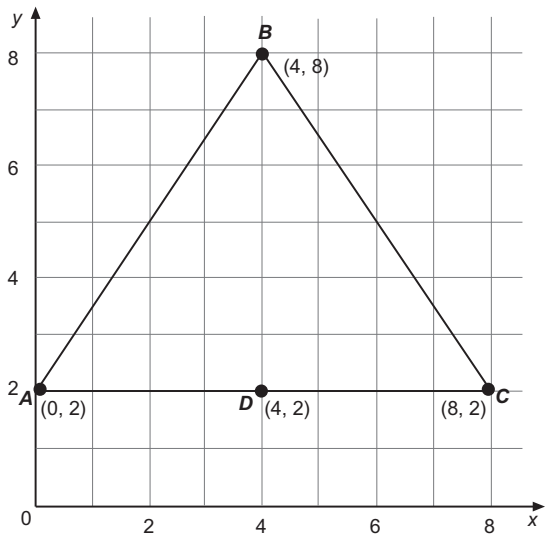
$$d = \sqrt{52}$$

$d = 2\sqrt{13}$ *or* 7.21 (to the nearest hundredth)

Practice

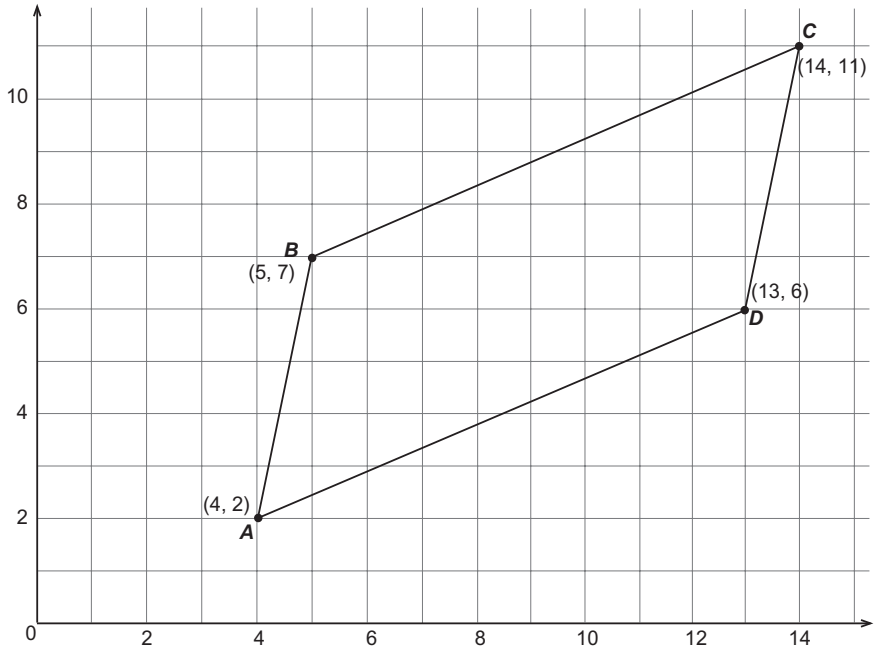
Complete the following.

Figure 1



1. Find the distance between points B and C (the length of side BC). Show your work.

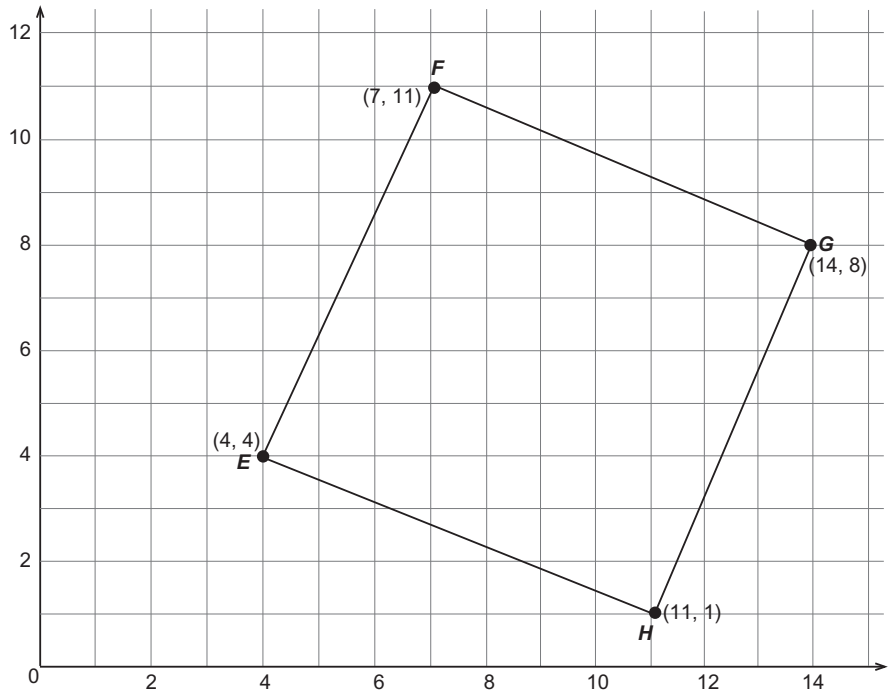
Figure 2



2. Find the distance between points *A* and *B* in parallelogram *ABCD*. Show your work.

3. Find the distance between points *B* and *C* in parallelogram *ABCD*. Show your work.

Figure 3



4. You previously verified that adjacent sides of Figure *EFGH* were perpendicular. You did this by determining the slope of each segment and verifying that the product of the slopes was -1. You will now verify that the sides are equal in measure by determining the length of each.

   a. The length of side *EF* is _____ . Show your work.

   b. The length of side *FG* is _____ . Show your work.

   c. The length of side *GH* is _____ . Show your work.

   d. The length of side *HE* is _____ . Show your work.

Practice

Use the list below to write the correct term for each definition on the line provided.

coefficient	**midpoint**	**slope**
hypotenuse	**parallel (∥)**	**square root (of a number)**
intersect	**perpendicular (⊥)**	***y*-intercept**
leg	**right triangle**	

_____ 1. the point on a line segment equidistant from the endpoints

_____ 2. a triangle with one right angle

_____ 3. a numerical factor in a term of an algebraic expression

_____ 4. in a right triangle, one of the two sides that form the right angle

_____ 5. one of two equal factors of a number

_____ 6. the longest side of a right triangle; the side opposite the right angle

_____ 7. being an equal distance at every point so as to never intersect

_____ 8. the value of y at the point where a line or graph intersects the y-axis

_____ 9. the ratio of change in the vertical axis (y-axis) to each unit change in the horizontal axis (x-axis) in the form $\frac{\text{rise}}{\text{run}}$

_____ 10. to meet or cross at one point

_____ 11. two lines that intersect to form a right angle