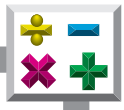


Lesson Four Purpose

- Associate verbal names, written word names, and standard numerals with integers, rational numbers, irrational numbers, and real numbers. (MA.A.1.4.1)
- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Understand that numbers can be represented in a variety of equivalent forms, including integers, fractions, decimals, percents, scientific notation, exponents, radicals, and absolute value. (MA.A.1.4.4)
- Understand and use the real number system. (MA.A.2.4.2)
- Add, subtract, multiply, and divide real numbers, including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)
- Relate the concepts of measurement to similarity and proportionality in real-world situations. (MA.B.1.4.3)
- Select and use direct (measured) and indirect (not measured) methods of measurement as appropriate. (MA.B.2.4.1)
- Solve real-world problems involving rated measures (miles per hour, feet per second). (MA.B.2.4.2)
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area, and volume and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)

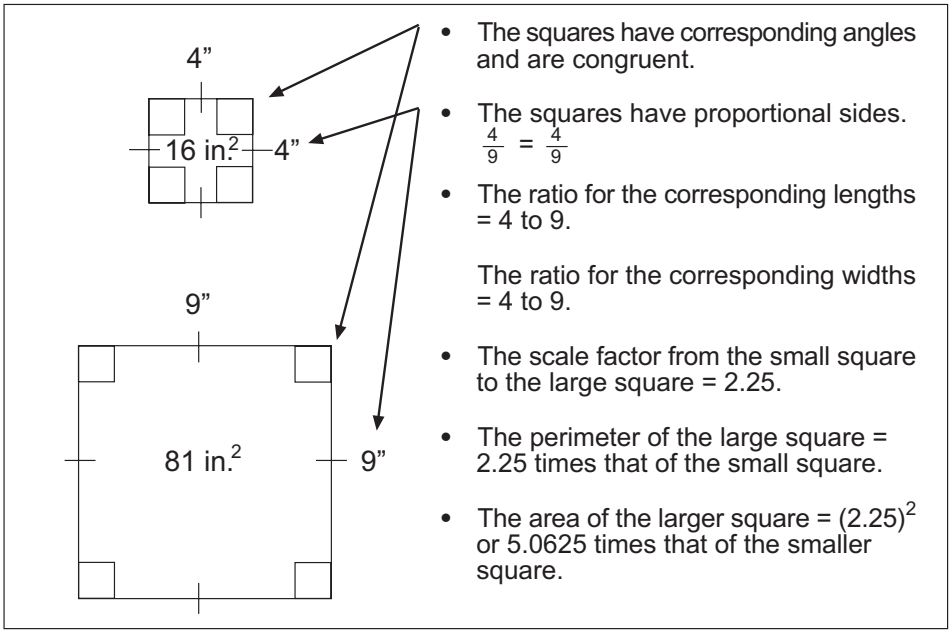
- Understand geometric concepts such as perpendicularity, parallelism, congruency, similarity, and symmetry. (MA.C.2.4.1)
- Represent real-world problem situations using finite graphs. (MA.D.2.4.1)
- Use equations and inequalities to solve real-world problems graphically and algebraically. (MA.D.2.4.2)

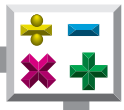
Applications of Ratio and Proportion in Similar Figures

You know that all the **angles (∠)** in a **square** are **right angles**, each measuring 90 **degrees (°)**. You also know that all the side lengths in a *square* are equal in length.

Suppose you have a square with a side length of 4 inches and another square with a side length of 9 inches. **Corresponding** (or matching) **angles** are **congruent (≅)** (the same shape and size) because they are all *right angles*. *Corresponding sides* are **proportional** because $\frac{4}{9} = \frac{4}{9}$. They have the same or constant ratio.

Comparing Two Similar Squares





The *ratio* for corresponding lengths (l) is 4 to 9, while the ratio for corresponding **widths (w)** is 4 to 9. Likewise, the ratio for length to *width* in the smaller square is 4 to 4, and the ratio for length to width in the larger square is 9 to 9.

The *scale factor* from the small square to the large square is 2.25 because the side length in the larger square is 2.25 times the length of the side length in the smaller square. The **perimeter (P)** (or distance around) of the large square will be 2.25 times the *perimeter* of the small square.

You know the **area (A)** (or region inside) of the small square is 16 square inches, and the *area* of the large square is 81 square inches. Area increases by the square of the scale factor and $(2.25)^2 = 5.0625$. The area of the larger square should be 5.0625 times the area of the smaller square. Does $16(5.0625) = 81$? Yes!

Practice

Match each definition with the correct term. Write the letter on the line provided.

_____ 1. figures or objects that are the same shape and size

_____ 2. the distance around a polygon

_____ 3. a one-dimensional measure of something side to side

_____ 4. a rectangle with four sides the same length

_____ 5. rays extending from a common endpoint called the vertex

_____ 6. common unit used in measuring angles

_____ 7. an angle whose measure is exactly 90°

_____ 8. the measure, in square units, of the inside region of a two-dimensional figure

_____ 9. the matching angles in similar figures

_____ 10. having the same or constant ratio

A. angle ($\angle$)

B. area (A)

C. congruent ($\cong$)

D. corresponding angles

E. degree (°)

F. perimeter (P)

G. proportional

H. right angle

I. square

J. width (w)

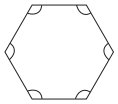
Working with Other Polygons

Polygons are closed-plane figures having at least three sides that are **line segments (—)** connected at their **endpoints**. **Similar figures (~)** have corresponding, congruent angles, and corresponding sides that are *proportional* in length. All **equilateral triangles** will be *similar*. The same will be true for *regular* **pentagons**, *regular* **hexagons**, and so on because all **regular polygons** are *equilateral* (all sides congruent) and *equiangular* (all angles congruent). The **sum** of the angles in any *pentagon* is 540 degrees and the angles in a regular pentagon are *congruent*. Side lengths in regular pentagons are congruent so we have the same situation as we did in the square.

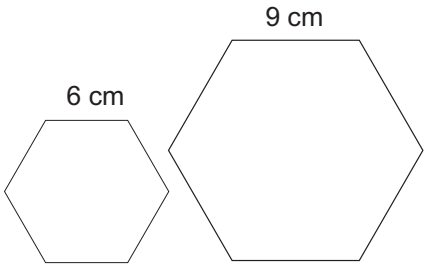
If the side lengths of a regular *hexagon* are 6 cm and the side lengths of another regular hexagon are 9 cm, the scale factor is 1.5. The perimeter of the large hexagon will be 1.5 times the perimeter of the small hexagon. Is 36 x 1.5 = 54? Yes. Area should increase by the square of that scale factor, $(1.5)^2$ or 2.25.

Comparing Two Similar Hexagons

A regular hexagon is equilateral (all sides the same length) and equiangular (all angles the same measure).



- The sum of the angles of a hexagon = 540°
- The side lengths and angles are congruent.



- The scale factor of the larger regular hexagon = 1.5.
- The perimeter of the larger regular hexagon = 1.5 times that of the smaller hexagon.
- The area of the larger hexagon = $(1.5)^2$ *or* 2.25 times that of the smaller hexagon.

Rectangles that are not squares may or may not be similar. Their corresponding angles will be congruent but they will be similar only if their corresponding sides are proportional.

Practice

Solve the following.

1. The dimensions of rectangle *ABCD* are 5 inches by 9 inches. If rectangle *EFGH* is to be similar and its short side is 17.5 inches, what should the length of the long side be?

   _____ inches

2. The *scale* for a map of the state of New York indicates that 1 inch = 11 miles. When a *line segment (—)* is drawn from Syracuse to Utica, its length is 4.5 inches. What is the distance from Syracuse to Utica in miles?

   _____ miles

3. The scale for a smaller map of New York is not provided but the map shows the distance in miles from Buffalo to Syracuse as 161 miles. When a *segment* is drawn between the two cities, its length is 6.4 centimeters. To the nearest mile, what does each centimeter represent on the map?

   _____ miles

4. The scale for a map of the state of Florida indicates that 1 inch = 16.5 miles. A segment drawn from Delray Beach to Vero Beach has a length of 5 inches. What is the distance in miles?

_____ miles

5. The scale for a smaller map of Florida is not provided but the map shows the distance in miles from Pensacola to Jacksonville as 364 miles. When a segment is drawn on the map between the two cities, its length is 7.1 centimeters. To the nearest mile, what does each centimeter represent on the map?

_____ miles

6. A basketball backboard and its supporting pole cast a shadow 120 centimeters long. A meter stick is held upright and **perpendicular (⊥)** to the ground nearby at the same time. The shadow of the stick measures 30 centimeters. What is the **height (*h*)** of the basketball pole?

Remember: 100 centimeters = 1 meter

_____ centimeters *or* _____ meters

7. A bank robbery was recorded on a surveillance camera. The robber is seen standing on one side of a counter. The height of the counter is known to be 45 inches. On the developed picture, the height of the counter is 2.5 inches. If the height of the robber on the developed picture is 4 inches, what is the actual height of the robber?

   _____ inches

8. A photocopy machine is being used to enlarge a verse written by a calligrapher on paper measuring 4 inches by 6 inches. If the 125% option is used on the machine, each of the dimensions will be 125% as long as the original.

   The dimensions of this enlargement will be

   _____ inches by _____ inches.

9. The dimensions for four rectangles are provided below. Two of the rectangles are similar. Which two are similar?

Dimensions of Four Rectangles

rectangle	length of rectangle in inches	width of rectangle in inches
A	4	5
B	9	10
C	3	3.75
D	5	6.5

Circle the letter of the correct answer.

a. rectangles *A* and *B* are similar
b. rectangles *A* and *C* are similar
c. rectangles *B* and *C* are similar
d. rectangles *B* and *D* are similar

10. The area of one square is 16 square feet and the area of a larger square is 400 square feet. What is the scale factor between the two squares?

scale factor = _____

Practice

Use the list below to write the correct term for each definition on the line provided.

equilateral triangle	**perpendicular (⊥)**	**scientific notation**
hexagon	**polygon**	**similar figures (~)**
pentagon	**regular polygon**	**standard form**

_____ 1. a method of writing the common symbol for a numeral

_____ 2. a polygon with five sides

_____ 3. a polygon that is both *equilateral* (all sides congruent) and *equiangular* (all angles congruent)

_____ 4. a polygon with six sides

_____ 5. two lines, two line segments, or two planes that intersect to form a right angle

_____ 6. figures that are the same shape, have corresponding, congruent angles, and have corresponding sides that are proportional in length

_____ 7. a triangle with three congruent sides

_____ 8. a closed-plane figure having at least three sides that are line segments and are connected at their endpoints

_____ 9. a shorthand method of writing very large or very small numbers using exponents in which a number is expressed as the product of a power of 10